Modelling Credit Risk

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Abstract

We review a manuscript of L.C. Rogers on credit risk modelling. The point of view in the manuscript is how one should model credit risk and the implementation of existing models. Since 1999 when it was written, a lot of new models and new developments have appeared. We'll discuss some of these models.

1 Introduction

Credit risk refers to the changes in credit quality of a security issuer or simply a counterparty. Firms finance their operations through equity and the issue of debt in the form often of bonds. A bond is simply a long-term debt. With a zero coupon bond, the firm promises to repay the face value of the bond at a certain time T, the maturity date. The price $b(t,T)$ at time T of such a bond at time t is given by $b(t,T) = e^{-y(t,T)(T-t)}$, where $y(t,T)$ is the yield of the bond. The price of a bond issued by the U.S. Treasury is $b_0(t,T) = e^{-r(t,T)(T-t)}$, where $r(t,T)$ is the risk-free rate. There's some chance that a corporation or a sovereign foreign government will not honour its obligations so investors charge them a higer interest rate, i.e. $y(t,T) > r(t,T)$. The difference $y(t,T) - r(t,T)$ is the credit spread, the premium required to hold an investment subject to risk. By term structure of credit spreads, we mean the correspondance between maturity dates T and the credit spreads. Firms are required by law to pay off their debts first before paying earnings to shareholders. When a firm stops servicing its obligations, we say that it defaults: in that case the bond holders take over the firm's assets. The terms default risk and credit risk are used interchangeably. Changes in the credit quality of a firm are indicators of default. The Moody's and Standard and Poor's give bond ratings for most traded bonds. These are judgments about firm's business and financial prospects. The best Moody's bonds are rated triple-A (Aaa), then come double-A (Aa) bonds. There are no fixed formula by which these ratings are calculated. Nethertheless, "since 1971, no bond that was initially rated triple-A by S & P's has defaulted in the year after the issue and fewer than one in a thousand has defaulted in the year after the issue. At the other extreme, over 2 percent of CCC bonds have defaulted in their first year and by year 10 about half have done so", [Brealey & Myers, 2001]. It's therefore not surprising that the credit spreads of a risky bond is related to the issuer's migrations through credit ratings classes.

We are interested in modelling two things, the probability of default and the price of a risky debt. Although given a model, one can deduce one from another it's not so clear how to calibrate or estimate parameters of models to answer these questions.

There are essentially two approaches to model credit risk: the intensity based approach in which default is seen as a perfectly unpredictable event and the structural approach in which default occurs when the firm's assets are insufficient to pay debt or fall below a prespecifed level. In the former category, we'll pay attention to the models which attempt to model credit ratings migrations of a firm and in the later we'll discuss the models which attempt to introduce jumps in the firm's assets dynamics.

2 Framework

We use a fixed probability space $(\Omega, \mathcal{H}, P)$, equipped with a filtration $(\mathcal{F}_t)_{t\geq 0}$ which describes the information flow over time. Each $\omega \in \Omega$ describes a possible state of the world. Let V_t, $(V_t(\omega))$ be the price of an asset for example a bond at time t with a promised payoff X at a terminal time T. $(V_t)_{t\geq 0}$ is assumed to be a $\mathcal{F}_t$-adapted stochastic process which means its value can be determined given the information available at time t. We have

$$V_t < E^P\left(Xe^{-r(t,T)(T-t)}\Big|\mathcal{F}_t\right), \tag{1}$$

where the right hand side is the present value of X. We have strict inequality otherwise no one will be interested in taking a risky position by holding the bond. (1) involves a conditional expectation under the probability measure P which describes the infinitesimals shocks affecting the asset price V_t. The values of V_t observed in the market will occur according to the probabilities given by the law of P. For pricing purposes, it would be easier to have

$$V_t = E^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}|\mathcal{F}_t\right), \tag{2}$$

where $\tilde{P}$ is a measure equivalent to P. Such a measure exists under the arbitrage-free condition. Arbitrage exist if an investor can guarantee a riskless return higher than the one given by the U.S. treasury by taking simultaneous positions in different assets. To get insight into the existence of an equivalent measure, notice that one can change the expectation of a discrete random variable $\sum x_i\ p_i$, by changing the probabilities p_i.

For a risky bond, default occurs at a random time τ which is modelled as a stopping time, i.e. we can determine based on the available information at time t whether default has occured or not. In general bond holders do not have all the available information so we let $\mathcal{G}_t$ be a filtration which encodes the information available to investors. In general it is known in advance what will be the payoff if default were to occur. For example a fraction L of the face value X will be lost. Let L be an $\mathcal{F}_t$-adapted stochastic process which models the recovery rate and let $I_{\{T<\tau\}}$ be the indicator function of the set $\{T < \tau\}$; The price $V(t,T)$ of a risky bond with terminal horizon T and payoff X is given by

$$V(t,T) = E^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}(I_{\{T<\tau\}} + (1-L_\tau)I_{\{T\geq\tau\}})|\mathcal{G}_t\right) \tag{3}$$

$$= E^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}|\mathcal{G}_t\right) - E^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}L_\tau I_{\{T\geq\tau\}}|\mathcal{G}_t\right). \tag{4}$$

The first term in the last equation is the price of a riskless zero-coupon bond with maturity T and face value X.

Models differ in the way they model the recovery process, the timing of default, i.e how default occurs and the choice of the filtration $\mathcal{G}_t$. In addition, so far we have implicitly assumed constant interest rates. Interest rates do change with time and the state of the economy. We would like to point out that not all combinations have been published.

Recovery process [Jarrow,Lando & Turnbull' 97] assume a constant recovery rate, $L(t) = 1-\delta$ for all t. [Duffie & Singleton' 95] consider a situation where the bond losses a fraction L_τ of its value at default. In [Jarrow & Turnbull' 98], on default the bond is replaced by $1-L_\tau$ riskless bonds with the promised payout.

Timing of default Let $(\mathcal{G}_t)_{t\geq 0}$ be the filtration generated by the asset price process V_t. In structural approaches, the default time τ is modelled as a $\mathcal{G}_t$-stopping time, i.e. it is perfectly predictable. This is certainly unrealistic in the short-run. Intensity based models, as mentioned, model the default time as an unpredictable event. In that case τ is certainly not measurable with respect to $\mathcal{G}_t$. It is assumed to be measurable in a larger filtration. In models which combine both approaches, the filtration is taken as the information available to public investors in form for example of noisy accounting reports (e.g. Enron), [Duffie & Lando' 01]. It is of interest in these models to determine if default can be predicted.

We recall that given a filtration $(\mathcal{H}_t)$, a stopping time τ is $\mathcal{H}$-predictable if there's an increasing sequence of $\mathcal{H}$-stopping time τ_n such that $\tau_n < \tau$ on $\tau > 0$ and lim $\tau_n = \tau$. We say that τ_n announces τ. A stopping time is $\mathcal{H}$-totally inaccessible if for any $\mathcal{H}$-predictable stopping time S we have

$$P\{\omega \in \Omega \ \tau(\omega) = S(\omega) < \infty\} = 0.$$

An inaccessible event is the mathematical formulation of a completely unpredictable event. An example of totally inacessible event is the first time a Poisson Process jumps. The jump of a Poisson process is uded below to model occurrence of default.

Models of interest rates The assumption of constant interest rates although unrealistic is often encountered in the literature. Other models of interest rates include a HJM framework e.g. [Jarrow & Turnbull' 95] and a Vasicek model e.g. [Jarrow & Turnbull' 98]. HJM stands for Heath-Jarrow-Morton (c.f [HJM' 98]). They modelled the entire term structure as a state variable providing conditions in a general framework that incorporates all the principles of arbitrage-free pricing and discount bond dynamics. In a Vasicek model, $r(0)$ is taken as deterministic and $r(t)$ satisfies the stochastic differential equation

$$\mathrm{d}r_t = \sigma \ \mathrm{d}W_t + \beta(r_\infty - r_t) \ \mathrm{d}t,$$

where W_t is a standard Brownian motion and σ, β, r_∞ are constants. This process can take negative values so a Cox-Ingerson-Ross model should be used, e.g. [Kim, Ramaswany, & Sundaresan' 93]. In the later model, $r(t)$ satisfies the SDE

$$\mathrm{d}r_t = -(\alpha - \sigma)\big(\delta(t) - \frac{\alpha\overline{\delta}}{\alpha - \sigma}\big)\ \mathrm{d}t - \sigma\sqrt{\delta(t)}\ \mathrm{d}W(t),$$

where $\alpha, \overline{\delta}, \sigma > 0$ and $\alpha > \sigma$.

3 Intensity based models

Using the Doob-Meyer decomposition, there exists an increasing $\mathcal{G}_t$-measurable process A with $A_0 = 0$ such that $I_{\{\tau \leq t\}} - A^\tau$ is a martingale, where $A^\tau_t = A_{t\wedge\tau}$ is the process A stopped at τ. If A is absolutely continuous with respect to the Lebesgue measure, i.e. if

$$A_t = \int_0^t h_s \mathrm{d}s,$$

for some non-negative process $h = (h_s)_{s>0}$, h_s also called intensity of τ can be interpreted as a hazard rate, i.e. as the instanteneous rate of default:

$$h_t = \lim_{h\to 0} \frac{1}{h} \mathrm{P}[\tau \in (t, t+h)|\mathcal{G}_t].$$

Under technical conditions, it can be shown that

$$\mathrm{P}(\tau > t) = \mathrm{E}[e^{-\int_0^t h_u\ \mathrm{d}u}].$$

Differentiating this relation gives us an expression for the density of τ:

$$\mathrm{P}(\tau \in \mathrm{d}t) = \mathrm{E}[h_t e^{-\int_0^t h_u\ \mathrm{d}u}].$$

We illustrate these results in the case h_s is identically constant in which case τ is modelled as the first time a Poisson process jumps. We then give pricing formulas under these conditions with a constant recovery rate and a fractional recovery rate. We finally apply this methodology to the case where the firm credit ratings change.

Case of a Poisson Process : $\mathbf{h_s} = \lambda$

We recall that a Poisson process $(N_t)_{t\geq 0}$ is a non-decreasing process with right-continuous path with values in $\mathbf{N}$ such that:

- $N_0 = 0$
- for any $0 \leq s_1 \leq t_1 \leq s_2 \leq t_2 \ldots \leq s_n \leq t_n$, the random variables $X_i = N(t_i) - N(s_i)$ are independent and the distributions of each X_i depends only on the length $t_i - s_i$
- for all $t \geq 0$, $N_t - N_{t^-}$ is either 0 or 1.

It can be shown that this definition uniquely determines a Poisson Process up to a parameter λ and that the increments X_i have a Poisson distribution with parameter $\lambda(t_i - s_i)$ for $s_i < t_i$, i.e.

$$\mathcal{P}(N_{t_i} - N_{s_i} = k) = \frac{1}{k!}\lambda^k (t_i - s_i)^k e^{-\lambda(t_i - s_i)}.$$

We let τ be the first time the Poisson Process N jumps, [Jarrow & Turnbull' 95] so $\{\tau > t\}$ implies $N_t = 0$. The occurence of default, state 1 is a disruption from the firm original state 0. We have:

$$\mathrm{P}(\{\tau > t\}) = \mathrm{P}(N_t - N_0 = 0) = e^{-\lambda t}.$$

This says that τ is exponentially distributed with density $f(t) = \lambda e^{-\lambda t}$. Now

$$\begin{aligned} \mathrm{P}(\tau \in (t, t+h) | \tau > t) &= \frac{\mathcal{P}(\tau \in (t, t+h))}{\mathcal{P}(\tau > t)} \\ &= \frac{f(t)h}{e^{-\lambda t}} = \lambda h, \end{aligned}$$

which implies that

$$\lim_{h \to 0} \frac{1}{h} \mathrm{P}(\tau \in (t, t+h) | \tau > t) = \lambda.$$

We now show that $I_{\{\tau \leq t\}} - \lambda t$ is a martingale. The underlying filtration $\mathcal{G}_t$ is the filtration generated by N. We have

- $N_t - \lambda t$ is $\mathcal{G}_t$-measurable
- $\mathcal{E}(|N_t|) = \mathcal{E}(N_t) = \lambda t$ since $N_t - N_0$ is Poisson distributed with parameter λt. It follows that $N_t - \lambda t$ is integrable.
- By the independent increment assumption, $N_t - N_s$ is independent of N_s for any $0 \leq s \leq t$, so

  $$\mathrm{E}(N_t - N_s | \mathrm{G}_s) = \mathrm{E}(N_t - N_s) = \mathrm{E}(N_t) - \mathrm{E}(N_s) = \lambda t - \lambda s.$$

  It follows that

  $$\mathrm{E}(N_t - \lambda t | \mathrm{G}_s) = \mathrm{E}(N_s - \lambda s | \mathrm{G}_s) = N_s - \lambda s.$$

Pricing formulas

We recall the expression (4) of the price of a risky bond

$$\begin{aligned} V(t,T) &= E^{\tilde{P}}\Big(Xe^{-r(t,T)(T-t)} | \mathcal{G}_t\Big) - E^{\tilde{P}}\Big(Xe^{-r(t,T)(T-t)} L_\tau I_{\{T \geq \tau\}} | \mathcal{G}_t\Big) \\ &= V_0(t,T) - E^{\tilde{P}}\Big(Xe^{-r(t,T)(T-t)} L_\tau I_{\{T \geq \tau\}} | \mathcal{G}_t\Big) \end{aligned}$$

We assume that $L_\tau = L$ is a constant. If $\tau < t$, then the price of the bond is $V(t,T) = (1-L)V_0(t,T)$, where $V_0(t,T)$ is the price of the risk-free bond. Now for $\tau > t$, there's still posssibility of default so

$$V(t,T) = V_0(t,T) - E^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}\int_t^T L(h_s e^{-\int_t^s h_u \ \mathrm{d}u})\mathrm{d}s|\mathcal{G}_t\right),$$

using the density of τ. This simplifies under the assumption $h_u = \lambda$ to

$$\begin{aligned} V(t,T) &= V_0(t,T) - LE^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}\int_t^T \lambda e^{-\lambda(s-t)}|\mathcal{G}_t\right) \\ &= V_0(t,T) - LE^{\tilde{P}}\left(Xe^{-r(t,T)(T-t)}(-e^{-\lambda(T-t)}+1)|\mathcal{G}_t\right) \\ &= V_0(t,T) - L(-e^{-\lambda(T-t)}+1)V_0(t,T) \\ &= (1-L+Le^{-\lambda(T-t)})V_0(t,T). \end{aligned}$$

We therefore write

$$V(t,T) = (1-L+Le^{-\lambda(T-t)}I_{\{\tau>t\}})V_0(t,T).$$

If $\tau > t$, the credit spread is simply given by

$$\frac{1}{T-t}\log\left(\frac{V_0(t,T)}{V(t,T)}\right) = \lambda - \frac{1}{T-t}\log(L+(1-L)e^{\lambda(T-t)}),$$

which is seen to be a decreasing function of $T-t$.

We now drop the assumption of constant interest rate and constant hazard rate and constant recovery rate. This situation is considered in [Duffie & Singleton' 95]; They found that the price of the bond at time $t < \tau$ is given by

$$V(t,T) = E[Xe^{-\int_t^T (r_s + h_s L_s)\mathrm{d}s}|\mathcal{G}_t],$$

where L_s is the fraction lost at default.

We show here how this result may be obtained. If the fraction lost on default were 1, the price would be for $t < \tau$

$$E[Xe^{-\int_t^T r_s \mathrm{d}s} I_{\{\tau > t\}} | \mathcal{G}_t] = E[Xe^{-\int_t^T (r_s + h_s)\mathrm{d}s} | \mathcal{G}_t].$$

Another way to interpret this result is that it is the price of a bond if 0 is received at default when the hazard rate is h_s. To receive a fraction $1 - L_s$ of the face value at default is equivalent from a pricing perspective to receive X with probability $1 - L_s$ and 0 with probability L_s. The later situation may be viewed as 0 recovery with hazard rate $h_s L_s$ which gives the result above.

Models for credit ratings migrations:

The use of credit ratings transition matrices in credit risk modelling started with the seminal work of Jarrow, Lando and Turnbull, [JLT' 97]. [Kijima & Komoribayashi' 98] improved the estimation of risk premia in [JLT' 97]. [Arvanitis, Gregory & Laurent' 99] provide a framework in which changes in ratings have memory. In the extension of [Das & Tufano' 96], the price of a risky bond does not depend only on the credit class.

The main objections to this model that remain are that there's no strong evidence that credit ratings transitions are Markovian. On the other hand it's hard to estimate the transition probabilities. The later problem has been adressed in [Israel, Rosenthal & Wei' 01]. We now describe the JLT model following [Kijima & Komoribayashi' 98].

JLT model: Let X_t represent the credit rating at time t of a bond issued by a firm. $(X_t)_{t \geq 0}$ is assumed to be a time-homogeneous Markov chain on the state space $M = \{1, 2, \dots K, K+1\}$, where state 1 represents the highest credit class, state K the lowest and state K+1 is default. For simplicity, state K+1 is assumed to be absorbing. Let $Q = (q_{ij}),\ i, j = 1 \dots K+1$ be the transition matrix of X, where

$$q_{ij} = P(X_{t+1} = j | X_t = i),\ i, j \in M,\ t = 0, 1, 2, \dots,$$

and P is the real word measure. Since state K+1 is absorbing

$$q_{K+1,K+1} = 1,\ q_{K+1,j} = 0,\ j = 1, \dots, K$$

For pricing purposes, we consider the corresponding Markov process $\tilde{X} = \{\tilde{X}_t, t = 0, 1, 2, \dots\}$ of credit rating under the risk-neutral probability $\tilde{P}$. $\tilde{X}$ needs not be Markovian and its transition probabilities are written

$$\tilde{q}_{ij} = \tilde{P}(X_{t+1} = j | X_t = i),\ i, j \in M,\ t = 0, 1, 2, \dots$$

We denote by $r(t)$ the default-free spot interest rate under $\tilde{P}$ and assume that the recovery rate δ is constant. $V_0(t, T)$ still denotes the price of the default free bond and $V_j(t, T)$ is the price of a bond in credit class j, $j \neq 0$. We also assume that $\tilde{X}$ and $r(t)$ are independent. This assumption is relaxed in the extension of [Lando' 98]. The probability of default given that the firm is in the credit class j is $\tilde{q}_{j,K+1}$. The price of a risky bond is

$$\begin{aligned} V_j(t, T) &= E\big[e^{-\int_t^T r(s)\mathrm{d}s}(I_{\{\tau_j > T\}} + \delta I_{\{\tau_j \leq T\}}) | X_t, \tau_j > t\big] \\ &= E\big[e^{-\int_t^T r(s)\mathrm{d}s} | X_t, \tau_j > t\big] E\big[I_{\{\tau_j > T\}} + \delta I_{\{\tau_j \leq T\}} | X_t, \tau_j > t)\big] \\ &= V_0(t, T)(\delta + (1 - \delta)\tilde{P}(\tau_j > T | X_t, \tau_j > t)). \end{aligned}$$

In general

$$\tilde{q}_{ij} = \pi_{ij}\ q_{ij},\ \ i, j \in M,$$

where the π_{ij} are the risk premium adjustments. In the JLT model, it is assumed that $\pi_{ij}(t) = \pi_i(t)$ for $j \neq i$. This leads to negative values of $\pi_j(0)$ when $q_{j,K+1}$ is sufficient small so Kijima and Komoribayashi assume $\pi_{ij}(t) = l_i(t)$ for $j \neq K+1$.

4 Structural models

The hazard rate models described in the previous section model the default as a completely unpredictable event. Default in reality is related to a financial data which may be related to traded data. From that point of view, it's certainly not completely unpredictable. However since the hazard rate is introduced ad hoc, it's always possible to calibrate it to fit economic data.

The structural models we describe in this section rely on the dynamics of the firm's assets. In these models, investors can observe the firm's assets and hence will not be surprised by default. Merton's model (cf. [Merton' 74]) is discussed below. To avoid this situation, attempts have been made to include jumps in the firm's financial assets value (cf. [Zhou' 97]). We'll show that with this model, default is still a predictable event. For a discussion of pros and cons of hazard rate and structural models, we refer to [Duffie & Singleton' 95], [Duffie & Singleton' 95] and the review papers of [Cooper & Martin' 96] and [Lando' 97].

Several stochastic differential equations are used in the literature to describe a firm's assets. [Merton' 74] used a log Brownian motion

$$\frac{\mathrm{d}V_t}{V_t} = \sigma \mathrm{d}W_t + r\mathrm{d}t,$$

with constant interest rate r and constant volatility σ. [Leland & Toft' 96] used

$$\frac{\mathrm{d}V_t}{V_t} = \sigma \mathrm{d}W_t + (r - \delta)\mathrm{d}t,$$

where r is the interest rate, σ a positive constant and δ the constant rate of dividends payed to shareholders. Another structural approach is the one of [Kim, Ramaswany, & Sundaresan' 93]. They used

$$\frac{\mathrm{d}V_t}{V_t} = \sigma \mathrm{d}W_t + (\alpha - \gamma)\mathrm{d}t,$$

for constants, σ, α and γ. We discuss Merton's model.

The Merton model:

We assume that the firm is financed by equity and bonds and is not allowed to pay dividends nor issue new debt equal or higher. Therefore at maturity the bondholders receive $\min\{V_T, K\} = K - \max(0, K - V_T)$ where K is the amount of debt. We let $\mathcal{G}_t = \sigma(V_s, s \leq t)$. The price of the risky bond at time $t < T$ is

$$V(t,T) = E[e^{-r(T-t)}(K - \max(0, K - V_T))|\mathcal{G}_t] = Ke^{-r(T-t)} - P(t, V_t, K),$$

where $P(t, V_t, K)$ is the value of a put option with strike price K and maturity T. The value of a call option $C(t, V_t, K)$ with the same characteristics is given by Black-Scholes call option formula

$$C(t, V_t, K) = N(d_1)V_t - N(d_2)Ke^{-r(T-t)},$$

where N is the cumulative normal distribution, V_t the value of the bond at time t and

$$d_1 = \log\frac{V_t/(Ke^{-r(T-t)})}{\sigma\sqrt{T-t}} + \sigma\sqrt{T-t}/2, \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}.$$

We next use the call-put parity:

$$C(t, V_t, K) - P(t, V_t, K) = V_t - Ke^{-r(T-t)},$$

so

$$\begin{aligned} V(t,T) &= (1 - N(d_1))V_t + N(d_2)Ke^{-r(T-t)} \\ &= N(-d_1)V_t + N(d_2)Ke^{-r(T-t)}. \end{aligned}$$

The credit spread is simply given by

$$\frac{1}{T-t}(\log\ (Ke^{-r(T-t)}) - \log V(t,T)) = \frac{1}{T-t}\Big(\log\ (Ke^{-r(T-t)}) - \log\big(Ke^{-r(T-t)}(\frac{1}{d}N(-d_1) + N(d_2))\Big),$$

where $d = V_t/Ke^{-r(T-t)}$. We see that the credit spread is

$$\frac{1}{T-t}[\log\big(\frac{1}{d}N(-d_1) + N(d_2)\big)]$$

A jump diffusion model:

In an attempt to simulate the possibility of default at maturity, [Zhou' 97] considered the following SDE

$$\frac{\mathrm{d}V_t}{V_t} = (\mu - \lambda v)\mathrm{d}t + \sigma\mathrm{d}W_t + (\Pi - 1)\mathrm{d}Y,$$

where μ, λ, v and σ are constants, Y is a Poisson process with intensity λ; $\Pi > 0$ is the jump amplitude with expected value equal to $v+1$. W, Y and Π are independent and Π is a i.i.d log-normal random variable such that $\ln(\Pi)$ has distribution $N(\mu_\pi, \sigma_\pi^2)$. The default time is set as $\tau = \inf\{t, V_t < V_B\}$. If we let, $\tau_n = \inf\{t, V_t \leq V_B + \frac{1}{n}\}$, there's a positive probability that τ_n converges to τ, since τ can cross the default boundary V_B via a continous crossing. Unless changes in V_t consist only of jumps, τ is totally inaccessible.

5 Integrating both approaches

The managers certainly know when default is imminent so the structural approach describes well how they view default. From the investors point of view, default comes by surprise so an intensity based approach should be used to describe how they experience default. [Duffie & Lando' 01] linked the two approaches by considering that investors receive noisy accounting reports $Y_{t_k} = V_{t_k} + U_{t_k}$ at discrete times $t_k, k = 1, \ldots n$, U_k being an independent noise random variable. [Kusuoka' 99] extends the work of Duffie and Lando to continuous time using continuous time filtering theory. More recently, [Cetin, Jarrow, Protter & and Yildirim' 02] have taken a different approach: They considered that bond investors have just less information.

6 Conclusion

There are other models which do not fit in the previous frameworks. In his manuscript, L.C. Rogers mention the works of [Hull & White' 95], [Beume, Hilberink, & Vellekoop' 98] and [Wong' 98]. The current research directions seem to be more complicated structural models with incomplete information or information reduction. Perhaps in these frameworks, one should consider the correlation of default of different firms, consider proportional losses at default and the pricing of convertible bonds. A path which seems not to have been taken is to use filtering theory for modelling credit ratings migrations.

References

[1] Arvanitis, A., Gregory, J. and Laurent, J.P., Building models for credit spreads, J. Derivatives, 6 (3), 27-43, 1999.

[2] Beumee, J.G.B. Hilberink, B.M. and Vellekoop, M.H. Valuation of contingent claims on a defaultable asset. Working paper, University of Twente, 1998.

[3] Brealey, R. and S. Myers, Principles of corporate finance, 6th edition, McGraw-Hill, New-York, 2001.

[4] Brzeźniak, Z. and Zastawniak, T. Basic stochastic processes, Springer, 1999.

[5] Cetin, U. Jarrow, R. Protter, P. and Yildirim, Y. Modeling credit risk with partial information, Manuscript, 2002.

[6] Cooper I and Martin, M. Default risk and derivative products. Applied Mathematical Finance, 3, 53-74, 1996.

[7] Cossin, D. and Pirotte, H. Advanced credit risk analysis, John Wiley & Sons, 2001.

[8] Das, S.R. and Tufano, P. Pricing credit-sensitive debt when interest rated, credit ratings, and credit spreads are stochastic, J. Financial Engineering, 5, 161-198, 1996.

[9] Duffie, D. and Lando, D. Term structures of credit spreads with incomplete accounting information, Econometrica, 69 (3), 633-334, 2001.

[10] Duffie, D. and Singleton, K. Econometric modeling of term structure of defaultable bonds. Working paper. Stanford University, 1995.

[11] Duffie, D. and Singleton, K. An econometric model of the term structure of interest rate swap yields. Working paper. Stanford University, 1995 and J. Finance 52, 1287-1321, 1997.

[12] Giesecke, K. Credit risk modelling and valuation: an introduction, Manuscript, 2002.

[13] Giesecke, K. Default compensator, incomplete information and the term structure of credit spreads, Manuscript, 2002.

[14] Jarrow, R.A. and Turnbull, S.M. Pricing derivative securities on financial securities subject to risk, J. Finance, 50, 53-85, 1995.

[15] Jarrow, R.A. Lando, D. , Turnbull, S.M. A Markov model for the term structure of credit risk spreads, Rev. Fin. Studies 10, 481-523, 1997.

[16] Jarrow, R.A. and Turnbull, S.M. The intersection of market and risk, Preprint, 1998.

[17] D. Heath, R. Jarrow and A. Morton, Bond pricing and the term structure of interest rates: A new methodology for contingent claim valuations, Econometrica, 60, 77-105, 1992.

[18] Hull, J. and White, A. The impact of default risk on the prices of options and other derivative securities, J. Banking and Finance 19, 299-322, 1995.

[19] Israel, R.B., Rosenthal, J.S. and Wei, J.Z. Finding generators for Markov chains via empirical transition matrices, with applications to credit ratings, Math Finance, 11 (2), 245-265, 2001.

[20] JeanBlanc, M. and Rutkowski, M. Modelling of default risk: An overview, Manuscript, 1999.

[21] Kijima, M. and Komoribayashi, K. A Markov chain model for valuing credit risk derivatives, J. Derivatives, 6 (1), 97-108, 1998.

[22] Kusuoka, S. A remark on default risk models, Advances in Mathematical Economics 1, 69-82, 1999.

[23] Lando, D. Modelling bonds and derivatives with credit risk, In M. Dempster and S. Pliska, editors,Mathematics of Financial Derivatives, pages 369-393. Cambridge University Press, 1997.

[24] Lando, D. On cox processes and credit risky securities, Manuscript, 1998.

[25] Neftci, S.N. An introduction to the mathematics of financial derivatives, Academic Press, 1996.

[26] L.C. Rogers, credit risk modelling, Manuscript, 1999.

[27] Rolski, T. Schmidli, H. Schmidli, V. and Teugels, J. Stochastic processes for insurance and finance, John Wiley & Sons, 1999.

[28] Ross, S. Stochastic Processes, John Wiley & Sons, 1983.

[29] Zhou, C A jump diffusion approach to modeling credit risk and valuing defaultable securities, preprint, 1997.