Math 181H: **Hour Exam 1** (Solutions)

Prof. S. Smith: February 18, 1992

Problem 1: If a radioactive element has a half-life of 810 years, how much of an original 10g is left after 300 years?

$$\ldots = (10g)2^{-\frac{300}{810}} \cong 7.736g$$

Problem 2: Find the derivative of $f(x) = \tan^{-1}(e^x)$.

$$\ldots = \frac{1}{1+(e^x)^2}e^x = \frac{e^x}{1+e^{2x}}$$

Problem 3: Find

$$\lim_{x\to 0} \frac{\cosh x - 1}{x^2}$$

By 0/0 l'H, $= \lim_{x\to 0} \frac{\sinh x}{2x}$*; again 0/0 l'H,* $= \lim_{x\to 0} \frac{\cosh x}{2} = \frac{1}{2}$

Problem 4: Find $\int x \csc^2 x\, dx$.

By parts: $u = x, dv = \csc^2 x\, dx$, *get* $du = dx, v = -\cot x$;
so $= -x\cot x + \int \cot x dx = -x \cot x + \ln|\sin x| + C$.

Problem 5: Evaluate $\int_4^\infty xe^{-x^2} dx$.

$$\ldots = -\tfrac{1}{2}e^{-x^2}|_4^\infty = -\tfrac{1}{2}(0 - e^{-16}) = \tfrac{1}{2e^{16}}.$$

Problem 6: Find $\int \tan^3 x \sec^3 x\, dx$.

Using $u = \sec x, du = \sec x \tan x\, dx, \tan^2 x = \sec^2 x - 1$,
get $= \int (u^2-1)u^2\, du = \int u^4\, du - \int u^2\, du = \frac{u^5}{5} - \frac{u^3}{3} = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$.

Problem 7: Find

$$\int \frac{dx}{\sqrt{x^2+2x+5}}$$

Notice completing square, $x^2 + 2x + 5 = (x+1)^2 + 2^2$.
So use $x + 1 = 2\tan u, dx = 2\sec^2 u\, du, \sqrt{x^2+2x+5} = 2\sec u$.
So $= \int \frac{2\sec^2 u\, du}{2\sec u} = \int \sec u\, du = \ln|\sec u + \tan u| = \ln|\frac{\sqrt{x^2+2x+5}}{2} + \frac{x+1}{2}| + C$
$= \ln|\sqrt{x^2+2x+5} + x + 1| + C'$ *(where* C' *is* $C - \ln 2$*).*

Problem 8: Find

$$\int \frac{x-11}{x^2+3x-4}dx$$

Note $x^2 + 3x - 4 = (x+4)(x-1)$, *so partial fractions* $\frac{A}{x+4} + \frac{B}{x-1} = \frac{x-11}{x^2+3x-4}$
gives $A(x-1) + B(x+4) = x - 11$, *and then* $A + B = 1, -A + 4B = -11$.
Then add to get $5B = -10$, *so that* $B = -2, A = 3$.
Now $= \int \frac{3}{x+4} - \frac{2}{x-1}\, dx = 3\ln|x+4| - 2\ln|x-1| = \ln\frac{|x+4|^3}{|x-1|^2} + C$.

Problem 9: Approximate $\int_1^3 x^4\, dx$ by Simpson's rule with $n = 4$ divisions.
(OK to leave answer in fractions).

Have $\Delta x = \frac{3-1}{4} = \frac{1}{2}$. *(Notice exact value is* $\frac{x^5}{5}|_1^3 = \frac{1}{5}(243-1) = 48\frac{2}{5}$*).*
So $\frac{1/2}{3}[1^4 + 4(\frac{3}{2})^4 + 2\cdot 2^4 + 4(\frac{5}{2})^4 + 3^4] = \frac{1}{6}[1 + \frac{81}{4} + 32 + \frac{625}{4} + 81] = \frac{1}{6}\frac{581}{2} = \frac{581}{12} = 48\frac{5}{12}$.