Math 181H: **Hour Exam 3** (Solutions)

Prof. S. Smith: 28 April 1992

Problem 1: Test for convergence (and evaluate if possible) $\sum_{n=0}^{\infty} \frac{2^n-1}{3^n}$.

Get $\sum(\frac{2}{3})^n - \sum(\frac{1}{3})^n = \frac{1}{1-\frac{2}{3}} - \frac{1}{1-\frac{1}{3}} = 3 - \frac{3}{2} = \frac{3}{2}$.

Problem 2: Test for convergence: $\sum_{n=1}^{\infty} \frac{1}{1+\ln n}$.

Can do comparison: for $n \geq 1$ *have* $n \geq 1 + \ln n$, *so* $\frac{1}{1+\ln n} \geq \frac{1}{n}$ = *harmonic; diverges.*

Problem 3: Test for convergence: $\sum_{n=0}^{\infty} \frac{n^n}{(3n+1)^n}$.

Root:$\lim_{n\to\infty}(\frac{n^n}{(3n+1)^n})^{\frac{1}{n}} = \lim_n \frac{n}{3n+1} = \lim_n \frac{1}{3+\frac{1}{n}} = \frac{1}{3} < 1$, *so converges.*

Problem 4: Test for convergence: $\sum_{n=0}^{\infty} \frac{(n+3)!}{3!n!3^n}$.

Ratio:$\lim_{n\to\infty} \frac{(n+4)!}{3!(n+1)!3^{n+1}} \frac{3!n!3^n}{(n+3)!} = \lim_n \frac{n+4}{3(n+1)} = \lim_n \frac{1+\frac{4}{n}}{3(1+\frac{1}{n})} = \frac{1}{3} < 1$, *so converges.*

Problem 5: Determine the interval of convergence of $\sum_{n=0}^{\infty} \frac{x^n e^n}{n+1}$.

Ratio: $\lim_{n\to\infty} \frac{x^{n+1}e^{n+1}}{n+2} \frac{n+1}{x^n e^n} = \lim_n xe\frac{n+1}{n+2} = xe$, *so converges for* $|x| < \frac{1}{e}$.

At $\pm\frac{1}{e}$ *get* $\sum_n \frac{(\pm 1)^n}{n+1}$. *For* $+\frac{1}{e}$*: harmonic, diverges; for* $-\frac{1}{e}$*: alt. harmonic, converges.*

So interval is $[-\frac{1}{e}, \frac{1}{e})$.

Problem 6: Determine the (Maclaurin) series at $a = 0$ for $\cos \frac{x}{\pi}$.

Derivs: $\cos \frac{x}{\pi}, -\frac{1}{\pi}\sin \frac{x}{\pi}, -\frac{1}{\pi^2}\cos \frac{x}{\pi}, \frac{1}{\pi^3}\sin \frac{x}{\pi}, \frac{1}{\pi^4}\cos \frac{x}{\pi}, \ldots$; *at* $a = 0$: $1, 0, -\frac{1}{\pi^2}, 0, \frac{1}{\pi^4}, \ldots$.

So series is $1 - \frac{1}{\pi^2}\frac{x^2}{2!} + \frac{1}{\pi^4}\frac{x^4}{4!} - \ldots = \sum_{n=0}^{\infty} \frac{(-1)^n}{\pi^{2n}} \frac{x^{2n}}{(2n)!}$.

Problem 7: Determine the Taylor series about $a = -1$ for $\frac{1}{x}$.

Derivs: $\frac{1}{x}, -\frac{1}{x^2}, \frac{2}{x^3}, -\frac{6}{x^4}, \ldots$;

values at $a = -1$ *are* $-1, -1, -2, -6, \ldots$.

So series is $-1 - (x+1) - 2\frac{(x+1)^2}{2!} - 6\frac{(x+1)^3}{3!} - \ldots = -\sum_{n=0}^{\infty}(x+1)^n$.

Problem 8: Determine the binomial series for $(1+x)^{\frac{3}{2}}$.

$1 + \frac{3}{2}x + \frac{3}{2}\frac{1}{2}\frac{x^2}{2!} + \frac{3}{2}\frac{1}{2}\frac{-1}{2}\frac{x^3}{3!} + \ldots + \frac{3}{2}\frac{1}{2}\frac{-1}{2}\ldots(\frac{3}{2} - n + 1)\frac{x^n}{n!} + \ldots$

Problem 9: Use a Taylor polynomial, based at $a = 0$ and up to x^5-term, to approximate the value of $\sin 1$; also estimate the error.

Approximately: $1 - \frac{1^3}{3!} + \frac{1^5}{5!} = 1 - \frac{1}{6} + \frac{1}{120} = \frac{101}{120}$.

Error at most $\frac{1}{7!} = \frac{1}{5040}$ *(so good to at least 3 decimal places).*