MATH 181 FINAL EXAM (Form A) SUMMER 1992

NAME(Print): ____
LAST FIRST

T.A.:__________________ SSN:______________________________________

DIRECTIONS

1. Place ID out for inspection.
2. Check to see that you have one complete exam. There are 20 multiple choice questions, a cover page and an answer sheet.
3. DO NOT DETACH THE ANSWER SHEET.
4. Blacken the circle of the correct answer on the answer sheet.
5. SHOW ALL WORK in the space provided by each problem.
6. Giving or receiving help on a final exam is grounds for dismissal from UIC.
7. Each problem is worth 10 points. No partial credit will be given.

Scoring : ______________ $\times$ 10 = ______________/200

1) The value of $\dfrac{d}{dx}\ln\big(\ln(\ln x)\big)$ at $x = e^3$ is:

A) $\dfrac{1}{\ln 3}$

B) $\dfrac{1}{\ln(\ln 3)}$

C) $\dfrac{e^3}{\ln(\ln 3)}$

D) $\dfrac{1}{3e^3 \ln 3}$

E) $\dfrac{1}{3\ln^2 x}$

2) Find $\displaystyle\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$

A) -1

B) $-\dfrac{1}{2}$

C) 0

D) $\dfrac{1}{2}$

E) ∞

3) Find the value of the integral $\displaystyle\int_0^{\pi/4} \tan^2 x \sec^4 x\, dx$.

A) $\dfrac{2}{15}$

B) $\dfrac{4}{15}$

C) $\dfrac{2}{5}$

D) $\dfrac{8}{15}$

E) $\dfrac{2}{3}$

4) Find the value of the integral $\int_1^2 \frac{1}{x^3+x}dx$.

A) $3\ln 2 - \ln 5$

B) $\frac{3\ln 2 - \ln 3}{2}$

C) $\frac{\ln 2 - 2\ln 3}{2}$

D) $\frac{\ln 2 - \ln 5}{2}$

E) $\frac{3\ln 2 - \ln 5}{2}$

5) $\int_0^2 \frac{x^3}{\sqrt{x^2+4}}dx$

A) $\frac{10}{3}(2+\sqrt{2})$

B) $\frac{16}{3}(5-2\sqrt{2})$

C) $\frac{8}{3}(2-\sqrt{2})$

D) $\frac{14}{3}(2\sqrt{2}-1)$

E) $\frac{20}{3}(4+\sqrt{2})$

6) Evaluate the improper integral $\int_1^\infty \frac{\ln x}{x^2}dx$.

A) 0

B) $\frac{1}{4}$

C) $\frac{1}{3}$

D) $\frac{1}{2}$

E) 1

7) Find the arc length of the curve $y^2 = x^3$ from $(0,0)$ to $(1/4, 1/8)$.

A) $65/216$

B) $29/108$

C) $59/216$

D) $37/108$

E) $61/216$

8) The curve $y = 1 - x^2 \quad 0 \leq x \leq 1$ is rotated about the y axis find the area of the surface.

A) $\frac{\pi}{3}(2\sqrt{2} + \sqrt{3})$

B) $\frac{\pi}{6}(5\sqrt{5} - 1)$

C) $\frac{\pi}{4}(\sqrt{5} - 2)$

D) $2\pi(2\sqrt{2} - 1)$

E) $\frac{\pi}{5}(\sqrt{5} - 1)$

9) The solution of the DIFFERENTIAL equation with the given condition $e^y y' = \dfrac{3x^2}{1+y}$; $y(2) = 0$ is:

A) $e^y = x^3 - 2$

B) $\frac{y^2}{2}e^y = x^3 - 1$

C) $e^{y^2} = 3x^2 + 8$

D) $ye^y = x^3 - 8$

E) $y^2e^y = x^3 - \frac{1}{3}$

10) Find the slope of the tangent line to the curve with parametric equations $x = 1 + \ln t$, $y = t^2 - 3t$ at the point where $t = 3$.

A) 9

B) 7

C) 5

D) 3

E) 1

11) Find the area inside one leaf of $r = \cos 2\theta$.

A) $\frac{\pi}{8}$

B) $\frac{\pi}{4}$

C) $\frac{\pi}{12} - \frac{\sqrt{3}}{16}$

D) $\frac{\pi}{24} - \frac{\sqrt{3}}{32}$

E) $\frac{\pi}{48} - \frac{\sqrt{3}}{64}$

12) The cartesian equation of the curve given by $x = \sec\theta,\ y = \tan\theta \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$ is:

A) $x^2 + y^2 = 1,\ x \geq 1$

B) $x = \sqrt{y^2 + 1},\ x \geq 1$

C) $x^2 + y^2 = 1,\ y \geq 1$

D) $y = \sqrt{1 - x^2},\ y \geq 1$

E) $x^2 - 2y^2 = 1,\ y \geq 2$

13) The graph shown can best be described by the equation

A) $r = 4 - 3\sin\theta$

B) $r = 4 + 3\cos\theta$

C) $r = 3(1 + \sin\theta)$

D) $r = 3 - 4\sin\theta$

E) $r = 2 + 3\cos\theta$

14) Indicate the reason for the convergence or divergence of

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - 1}$$

A) Divergent by comparison with $\Sigma_{n=2}^{\infty} \dfrac{1}{n}$

B) Convergent by comparison with $\Sigma_{n=2}^{\infty} \dfrac{1}{n^{\frac{3}{2}}}$

C) Convergent by integral test

D) Convergent by ratio test

E) Divergent by ratio test

15) Indicate the reason for the convergence or divergence of

$$\sum_{n=1}^{\infty} n e^{-n^2}$$

A) Divergent by limit comparison test

B) Divergent by comparison with $\Sigma_{n=1}^{\infty} \dfrac{1}{n}$

C) Convergent by comparison with $\Sigma_{n=1}^{\infty} \dfrac{1}{n}$

D) Divergent by integral test

E) Convergent by integral test

16) Indicate the reason for the convergence or divergence of

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$$

A) Convergent by integral test

B) Convergent by ratio test

C) Divergent by ratio test

D) Convergent by comparison with $(-3)\Sigma_{n=1}^{\infty} \frac{1}{n^3}$

E) Divergent by comparison with $\Sigma_{n=1}^{\infty} \frac{1}{n^2}$

17) The Taylor series for $f(x) = \frac{1}{x}$ about $c = 1$ is given by

A) $\Sigma_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$

B) $\Sigma_{n=0}^{\infty} (-1)^n x^n$

C) $\Sigma_{n=0}^{\infty} (-1)^n n!(x-1)^n$

D) $\Sigma_{n=0}^{\infty} (-1)^n (x-1)^n$

E) $\Sigma_{n=0}^{\infty} \frac{(-1)^n}{n!} (x-1)^n$

18) A power series for $\int_0^x \frac{t}{1-t} dt$ about $c = 0$ is given by

A) $\Sigma_{n=2}^{\infty} \frac{1}{n+1} x^{n+1}$

B) $\Sigma_{n=2}^{\infty} \frac{1}{n} x^n$

C) $\Sigma_{n=2}^{\infty} x^n$

D) $\Sigma_{n=1}^{\infty} x^n$

E) $\Sigma_{n=0}^{\infty} \frac{(01)^n}{(2n+1)!} x^{2n+1}$

19) Find the first four terms in the Maclaurin series for $f(x) = xe^{-x}$.

A) $x - x^2 + x^3 - x^4$

B) $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4$

C) $x - x^2 + \frac{1}{2}x^3 - \frac{1}{6}x^4$

D) $x - 2x^2 + 3x^3 - 4x^4$

E) $x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4$

20) Find the radius of convergence of $\sum_{n=1}^{\infty} \frac{(x+3)^n}{\sqrt{n}\ 2^n}$.

A) 1/2

B) 1

C) 2

D) 3

E) 4