Math 181H: **Practice Exam 1** NAME: ____________________

(was Exam2 in old Math 131, Feb 1990) SS# ____________________

Problem 1: Find $F'(t)$ if $F(t) = \arccos\sqrt{2t-1}$. $\ldots - \frac{1}{\sqrt{1-(\sqrt{2t-1})^2}}\frac{1}{2\sqrt{2t-1}}2 = \frac{-1}{\sqrt{(2-2t)(2t-1)}}$

Problem 2: Find $\int_0^1 x^2 e^{-x} dx$.

By parts with $u = x^2, dv = e^{-x}dx$, *get* $du = 2xdx, v = -e^{-x}$, *so* $[-x^2e^{-x} + 2\int_0^1 xe^{-x}dx]_0^1$.

By parts again with $u = x, dv = e^{-x}dx$, *get* $du = dx, v = -e^{-x}dx$,

so $= [-x^2e^{-x} + 2(-xe^{-x} + \int e^{-x}dx)]_0^1 = [-x^2e^{-x} - 2xe^{-x} - 2e^{-x}]_0^1$

$= (-\frac{1}{e} - 2\frac{1}{e} - 2\frac{1}{e}) - (0 + 0 - 2) = 2 - \frac{5}{e}$.

Problem 3: Find $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^2 x}dx$.

Subst: $u = \cos x$ *so* $du = -\sin x$, *to get* $-\int_0^{\frac{\pi}{4}} \frac{du}{u^2} = u^{-1}|_0^{\frac{\pi}{4}} = \frac{1}{\sqrt{2}/2} - \frac{1}{1} = \sqrt{2} - 1$.

Problem 4: Find $\frac{dy}{dx}$ if $y = e^{(e^x)}$.

Via logarithmic diff: $\ln y = e^x$, *so* $\frac{y'}{y} = e^x$, *hence* $y' = ye^x = e^{e^x}e^x = e^{e^x+x}$.

Problem 5: Find $\frac{dy}{dx}$ if $y = \frac{(x+1)^4(x+5)^3}{(x-3)^8}$. (Hint: logarithmic differentiation)

$\ln y = 4\ln(x+1) + 3\ln(x+5) - 8\ln(x-3)$, *so* $\frac{y'}{y} = \frac{4}{x+1} + \frac{3}{x+5} - \frac{8}{x-3}$,

hence $y' = [\frac{(x+1)^4(x+5)^3}{(x-3)^8}][\frac{4}{x+1} + \frac{3}{x+5} - \frac{8}{x-3}]$.

Problem 6: A bacteria culture starts with 500 and after 3 hours there are 8000. Find an expression for the number $P(t)$ of bacteria after t hours.

Given $8000 = 500e^{3k}$, *so* $16 = e^{3k}$, *hence* $\ln 16 = 3k$, *or* $k = \frac{\ln 16}{3}$.

Thus $P(t) = 500e^{\frac{\ln 16}{3}t} = 500(16)^{\frac{t}{3}}$.

Find the number of bacteria after 4 hours. $\ldots 500(16)^{\frac{4}{3}}$

When will the population reach 20,000?

$20000 = 500e^{\frac{\ln 16}{3}t}$, *so* $40 = e^{\frac{\ln 16}{3}t}$; *so* $\ln 40 = \frac{\ln 16}{3}t$, *or* $t = 3\frac{\ln 40}{\ln 16}$.

Problem 7: Find $\int \ln\sqrt{x}dx$.

By parts, $u = \ln\sqrt{x}, dv = dx$ *gives* $du = \frac{1}{\sqrt{x}}\frac{1}{2\sqrt{x}} = \frac{dx}{2x}, v = x$.

So $x\ln\sqrt{x} - \frac{1}{2}\int dx = x(\ln\sqrt{x} - \frac{1}{2}) + C$.

Problem 8: Find $\int \frac{dx}{x^2\sqrt{16x^2-9}}$.

Use trig subst: $x = \frac{3}{4}\sec u, dx = \frac{3}{4}\sec u\tan u\, du, \sqrt{x^2 - \frac{9}{16}} = \frac{3}{4}\tan u$.

Then $= \int \frac{\frac{3}{4}\sec u\tan u\, du}{(\frac{9}{16}\sec^2 u)(3\tan u})) = \frac{4}{9}\int\cos u\, du = \frac{4}{9}\sin u = \frac{4}{9}\frac{\sqrt{x^2-\frac{9}{16}}}{x} = \frac{1}{9}\frac{\sqrt{16x^2-9}}{x} + C$.

Problem 9: Find $\int \frac{x+1}{x^2-x}dx$.

Partial fractions: From $A(x-1)+Bx = x+1$ *have* $A+B = 1, -A = 1$ *so* $A = -1, B = 2$.

Then $= -\int\frac{dx}{x} + 2\int\frac{dx}{x-1} = -\ln x + 2\ln(x-1) = \ln\frac{(x-1)^2}{x} + C$.

Problem 10: Find $\int csc^3 x dx$.

By parts: With $u = \csc x, dv = \csc^2 x$ *have* $du = -\csc x\cot x dx, v = -\cot x$.

So integral $I = -\csc x\cot x - \int\cot^2 x\csc x dx = -\csc x\cot x - \int(\csc^2 x - 1)\csc x dx$

$= -\csc x\cot x - \int\csc^3 x dx + \int\csc x dx = -\csc x\cot x - I - \ln|\csc x + \cot x|$.

Then $2I = -\csc x\cot x - \ln|\csc x + \cot x|$, *so* $I = -\frac{1}{2}(\csc x\cot x + \ln|\csc x + \cot x|)$.