M210 Spring 93 Final Exam May 4, 1993

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1. (20) i) Find the equation of the plane passing through the three points

$$(0, 1, 2), (1, -1, 2), (2, 1, 0).$$

Express your answer in the form $ax + by + cz = d$.

Ans: _____x+ _____y+ _____z = _____

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1. ii) Find the angle that the diagonal of a cube makes with one of its edges.

Ans:_____

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2. (15) Show that the equation of the tangent plane to the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

at the point (x_0, y_0, z_0) has equation

$$\frac{xx_0}{a^2} + \frac{yy_0}{b^2} + \frac{zz_0}{c^2} = 1$$

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3. (15) The circular helix $\vec{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$ intersects the curve $\vec{s}(t) = e^t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$ at the point $(1, 0, 0)$. Find the angle of intersection of the two curves.

Ans: _____

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4. (20) i). Let $z = \dfrac{x}{1+\cos(y)}$. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

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4. ii) Let $z = uve^u$ where $u = x^2 - y^2$ and $v = 2xy$. Find $\dfrac{\partial z}{\partial y}$.

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5. (20). Consider

$$\int_0^2 \int_{y^2}^{6-y} f(x,y)\,dx\,dy.$$

i) Sketch the region of integration.

ii) Change the order of integration.

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6. (20) Evaluate

$$\iiint_B \frac{1}{1+x^2+y^2+z^2}\,dV$$

where B is the ball of radius 1 centered at $(0,0,0)$:

$$B = \{(x,y,z) | x^2+y^2+z^2 \leq 1\}.$$

Ans: _____

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7. (15) Evaluate

$$\int_0^1 \int_0^{\sqrt{1-x^2}} e^{x^2+y^2}\, dy\, dx$$

by converting to polar coordinates.

Ans: _____

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8. (20) Consider the portion of the surface $z = xy$ which lies within the cylinder $x^2 + y^2 = 1$.

i) Write down (at least one) integral which gives the surface area.

ii) Evaluate the integral to find the surface area.

Ans: _____

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9. (20) Find the absolute (overall) maximum value of the function

$$f(x, y, z) = xy + z$$

subject to the constraint

$$x^2 + y^2 + z^2 \leq 1.$$

You must find the critical point(s) in $x^2 + y^2 + z^2 < 1$ and the maximum value(s) on $x^2 + y^2 + z^2 = 1$.

The overall max value occurs at _____ where $f(x, y.z) =$ _____

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10. (20) Let D be the upper half of the semicircle $x^2 + y^2 \leq 1$. Find $\bar{y}$, the y–coordinate of the centroid of D. You may assume that the area is $\frac{\pi}{2}$.

Ans: $\bar{y} =$ _____

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11. (15) Write the integral

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{+\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} (x^2+y^2+z^2)xy\,dz\,dy\,dx$$

as an integral in spherical coordinates. **You need not evaluate the integral!**