Here we give an example of solving a Cauchy-Euler nonhomogenous equation:
\[ x^2y''(x) + 3xy'(x) - 8y(x) = x + x^3; x > 0. \]

First we consider a trial solution of the form \( x^r \) and compute \( r \).
\[ y(x) = x^r, \quad y'(x) = rx^{r-1}, \quad y''(x) = r(r-1)x^{r-2}. \]

Substituting into the homogenous form of the equation gives
\[ r(r-1)x^r + 3rx^r - 8x^r = 0 \]

Next dividing by \( x^r \) gives a quadratic equation for \( r \).
\[ r(r-1) + 3r - 8 = r^2 + 2r - 8 = (r - 2)(r + 4) = 0. \]

The fundamental solution set is \( \{x^2, x^{-4}\} \).

Next we seek the particular solution by the method of undetermined coefficients:
\[ y_p(x) = Ax^3 + Bx^2 + Cx + D. \]

Substituting into the ODE gives
\[ 3 \cdot 2Ax^3 + 2Bx^2 + 3(3Ax^3 + 2Bx^2 + Cx) - 8(Ax^3 + Bx^2 + Cx + D) = x + x^3. \]

Equating coefficients of like powers of \( x \) to zero gives:
\[ A = 1/7, B = 0, C = -1/5, D = 0. \]

The total solution is now:
\[ y(x) = c_1x^2 + c_2x^{-4} + \frac{1}{7}x^3 - \frac{1}{5}x. \]

If initial conditions are given, then you would need to use this \( y(x) \) to find \( c_1 \) and \( c_2 \).