Math 220: EXAM II, 11/19/99

Be sure to show all work in the answer book. Indicate where you have used a TI-89 to get an answer. Please put a \boxed{ } around all final answers. The more work you show, the more partial credit you may get. Indicate your TA Section time as 10:00 or 12:00.

1. (a) Show that the solution \( h(t) \) to

\[
ay'' + by' + cy = \delta(t), \quad y(0) = y'(0) = 0
\]

is the same as

\[
ay'' + by' + cy = 0, \quad y(0) = 0, \quad y'(0) = 1/a
\]

(b) Write a convolution integral using \( h(t) \) which solves

\[
ay'' + by' + cy = \sin(t), \quad y(0) = y'(0) = 0
\]

2. Find \( y(t) \) for the initial value problem: (\( u(t) \) is the Heaviside function.)

\[
y' + 6y = [1 - u(t - 2)]e^{-6t}, \quad y(0) = 1
\]

3. Using the power series method, find the first four non zero terms in the series solution about \( x = 0 \) of

\[
y' + 2xy = 0, \quad y(0) = 1
\]

4. Find \( x(t) \):

\[
x' - y' = (\sin t)u(t - \pi), \quad x(0) = 1
\]
\[
x + y' = 0, \quad y(0) = 1
\]

5. Consider the following equation for the damped oscillator with displacement \( y(t) \):

\[
y'' + 10y' + 25y = u(t - 4\pi) \cos(2(t - 4\pi)), \quad y(0) = 1, \quad y'(0) = 0.
\]

(a) Is this SHM underdamped, overdamped, or critically damped?

(b) Make a rough sketch of the solution for \( 0 \leq t \leq 4\pi \).

(c) Make a rough sketch of the solution for \( 10\pi \leq t \leq 12\pi \):