Background: An important tool for forest management is the firebreak. Firebreaks are wide clear-cut paths through a section of forest that divide the section into smaller sections of forest called stands. The purpose of the firebreaks is that in the event of a fire starting in the forest, the damage will be confined to the stand in which the fire originates, thereby limiting the damage of the fire. Of course, the creation of firebreaks destroys a part of the forest, and so a general problem is to find an arrangement of firebreaks throughout the section of forest that on the one hand provides protection in the case of a fire but on the other hand does not cause the destruction of too much forest.

The assignment: You work as an analyst for AAA Forest Industries Corporation. A real estate holding company called BBB has requested a bid for the clearing of a system of firebreaks for a parcel of land that BBB holds in a Canadian forest. Your job is to write up a proposal to give to BBB that describes the arrangement of firebreaks that could be used, why each particular arrangement is proposed, and what AAA will charge BBB for cutting each arrangement. The proposal will consist of two parts: a letter to Mr. Green, the project manager at BBB, that describes in very general terms the proposed arrangements of the firebreaks, a brief explanation of the arrangements, and an explicit statement of what AAA with charge BBB for the construction of the firebreaks. The second part will be a technical report that provides a detailed mathematical justification and description of the proposed arrangements in the bid.

The outline is due in class on **Monday 18 February or Wednesday 20 February**. The first draft is due the following week in class. The final version of the essay is due Monday 10 March and Wednesday 12 March.

The letter to Mr. Green should very briefly state what the proposal is for. It should give a description of the pattern of the firebreaks and the number of firebreaks in the pattern, and the amount of money AAA will charge BBB for cutting these firebreaks. There are to be two proposals described in the letter to Mr. Green:
(1) One proposal is when the only consideration is to minimize total area of forest lost in the event of a fire. That is, minimize the sum of the area of forest in one stand of trees and the total area of all the firebreaks.
(2) The second proposal is when the only consideration is to minimize the revenue lost by the destruction of one stand of trees due to fire and due to the clearing of the firebreaks together with the cost to BBB for having AAA create the firebreaks.

The technical memo should explain how you obtained the proposed patterns and a justification for them. You should find an explicit function of n, the number of firebreaks, that expresses the quantity in each of the two proposals that is to be minimized. Use calculus to find the value of n that gives the minimum for a given general pattern. Further computations should be given that provide the value of each function for the optimal value of n. The memo should also contain explanations of what you are doing. It may also contain charts, tables or graphs, but these are optional.

Audience: You may assume Mr. Green is familiar with firebreaks and forest management. He is also an intelligent person with good common sense, but he does not know any mathe-

matics beyond a high school level. Your letter to Mr. Green should explain, in general terms, why an optimal number of firebreaks exists for each of the two proposals and state that the analysts at AAA have mathematically determined what these optimal values are. The letter should not go into the methods used to determine these optimal values. Mr. Green does not know any calculus. A challenge in writing the letter to Mr. Green is not to be so technical that you intimidate or confuse him but not to be so simplistic that you insult him.

The audience for the technical report is a person who is also familiar with firebreaks and forest management but is also someone who is very familiar with calculus and computational mathematics. This person knows what a function is, how to take derivatives of functions, and how to find global minima. The technical report can include mathematical notation as needed.

Specifics: The bid is for making the firebreaks on a 4 mile by 6 mile rectangular forested plot of land owned by BBB. You may assume that the generally accepted pattern for firebreaks for a rectangular parcel of land is to have n equally spaced firebreaks, each of the same width, all running parallel to the short sides of the rectangle. These n firebreaks include one firebreak on each of the short ends of the rectangular parcel. The diagram below illustrates $n = 3$ firebreaks. In this arrangement, if there are n firebreaks, then they are placed so that the entire plot is divided into $n - 1$ stands of trees, all of equal area. The number n of firebreaks is to be a whole number, so if the analysis give an optimal value of n that is not a whole number, then an appropriately chosen whole number value for n should be used in writing up the bid. Standard policy of AAA is that the firebreaks all have the same width, which is .01 mile. AAA will charge BBB $15,000 per square mile for cutting and clearing the firebreaks. The expected revenue for BBB from eventually harvesting the forest is $40,000 per square mile.
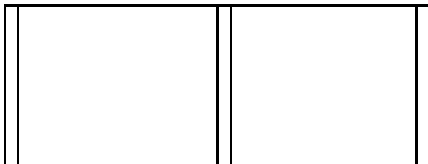
Additional comments: The letter to Mr. Green is a standard business letter and should be written as such. The letter should include the bid that AAA is making for each of the two proposals. That is, the bid is the amount of money AAA will charge BBB for making the firebreaks for each of the two proposals. The technical report should use elementary calculus to find the global minimum for an appropriate function for each proposal. Carefully describe the domain of the functions for which the global minimum is being computed. Note that the number of firebreaks must be an integer, and so may differ from the value of the independent variable that provides the global minimum of the function.