Turn in the following five problems Friday 8 February

B1: Let $L$ be an algebraic lattice and suppose $a, b, c \in L$ with $c$ compact, $a < b$, $c \leq b$ but $c \not\leq a$. Prove that there exists an element $m \in L$ with $a \leq m \leq b$ and $m$ is maximal with the property that $c \not\leq m$.
[Hint: You might want to use Zorn’s Lemma here. A statement of it is item (e) in the Preliminaries section of Burris and Sankappanavar. A more robust version is the following: If $\langle P, \leq \rangle$ is a nonvoid partially ordered set in which every chain has an upper bound, then $\langle P, \leq \rangle$ has a maximal element $m$, (that is, $m \in P$ and if $m \leq x \in P$, then $m = x$).]

B2: Show that if $L$ is an algebraic lattice and $a < b$ in $L$, then there exist elements $r$ and $s$ such that $a \leq r \prec s \leq b$.
[Hint: If you wish you may use B1 in your argument.]

Problem #8, Section I.4. To the question if $a \wedge b$ is always compact, either give a proof if the answer is yes or a counterexample if the answer is no.

Problem #7, Section I.5

B3: Give an example of two algebras $A$ and $B$ each of type $\mathcal{F}$ for which there is a one-to-one homomorphism $\alpha : A \to B$ and a one-to-one homomorphism $\beta : B \to A$, but $A$ and $B$ are not isomorphic.