Turn in the following five problems Monday 25 February.

B1: Prove the following representation theorem: Every finite distributive lattice is isomorphic to a ring of sets.
[Recall that a ring of sets is a family $\mathcal{F}$ of sets such that for every $X$ and $Y$ in $\mathcal{F}$, the sets $X \cap Y$ and $X \cup Y$ are in $\mathcal{F}$. You may want to use the fact proved in a previous homework exercise that every element in a finite lattice is the join of the join-irreducible elements below it.]

B2: Prove the following representation theorem: Every finite distributive lattice is isomorphic to the subuniverse lattice of a finite algebra having only unary and nullary operations.
[Note: An algebra $A$ has one or more nullary operations if and only if $\emptyset$ is not a subuniverse of $A$.]

B3: Let $A = \langle A, F \rangle$ be an arbitrary algebra. Prove that there exists an algebra $U = \langle A, G \rangle$ with every operation in $G$ being unary and $\text{Con} A = \text{Con} U$.

Problem #9 in II.5. In class we showed that the relation $\leq$ defined here is actually a partial order so you may omit this part of the problem. But do describe the quotient semilattices $S/\theta^a$. Also prove that for every $S$ and every $c \neq d$ in $S$, there exists an element $a \in S$ such that $(c, d) \not\in \theta^a$, that is, the congruences $\theta^a$ separate the elements of $S$.

Problem #3 in II.6. Note that this is an if and only if statement.