

Math 160 – Final Exam practice problems  
Fall 2012 – Brian Powers TA

1. Brachiosaurus femurs that have been discovered have an average length of 1916.5 mm and a standard deviation of 242 mm. Pierre the Paleontologist found a Brachiosaurus fossil. What is the probability its femur is longer than 2500mm?

2. In a survey of customers, a local pizza place found that 48 customers enjoy thin crust, 39 like extra toppings, 23 like both and 12 like neither. How many customers were surveyed?

3. Maximize  $7x-2y$  subject to: 
$$\begin{cases} 3x+2y \geq 6 \\ 2y-3x \leq 6 \\ x+4y \leq 26 \\ y \geq 0, x \leq 10 \end{cases}$$

4. Find the inverse of the following matrices: a)  $\begin{bmatrix} 5 & 3 \\ -6 & 4 \end{bmatrix}$  b)  $\begin{bmatrix} a & a+1 \\ a+2 & a+3 \end{bmatrix}$  c)  $\begin{bmatrix} 3 & 4 \\ k & 5 \end{bmatrix}$

For part b), is there any value of a for which there is NO inverse?

Is there any value of k for which part c) has no inverse?

5. A car manufacturer has plants in Knoxville and Detroit, with showrooms in Chicago and St. Louis. The Knoxville plant has 45 new cars, and Detroit has 33. Chicago needs 27 new models and St. Louis needs 18. Shipping cost from Knoxville is \$45/car to Chicago and \$34/car to St. Louis, while shipping from Detroit is \$30/car to Chicago and \$52/car to St. Louis. If the company wants to minimize costs: Letting  $x$ =# cars shipped from Knoxville to Chicago and  $y$ =#cars shipped from Knoxville to St. Louis:

a) State the objective function and all restriction inequalities.

b) Show the graph of the feasible set

c) Give the corner point coordinates

d) Find the solution, and state the minimum cost attained.

6. The Octomom has all her kids' socks in a big hamper. If there are 46 red socks and 63 blue socks in the hamper, and she just grabs enough socks for her eight kids without paying attention to the colors, what is the probability at least 5 of the kids can wear matching red socks?

7. The bipolar bear's mood can change every 5 minutes. When ferocious, 90% of the time it is calm 5 minutes later. If it is calm it may burst into a rage 5 minutes later with probability 80%. If I observed it 20 minutes ago and it was calm, what is the probability that it is currently in a rage?

In the long run, what proportion of the time does the bipolar bear spend in a ferocious rage?

8. When I make a math joke,  $2/7$  of the time the students laugh. Over the course of the semester I cracked 230 math jokes. Approximating with the normal distribution:

a) What is the probability that I got laughs at least 100 times?

b) What is the probability I got laughs at least 80 times but no more than 92 times?

9. Angstown has two exports: Suffering and Art. It requires \$0.50 worth of suffering and \$0.10 worth of art to generate \$1 of Art, and to create \$1 worth of suffering it only takes \$0.20 worth of suffering. If the town of Naïveille places an order of \$500 of Suffering and \$1,000 of Art, how much Suffering and Art must be produced in Angstown to meet the demands?

10. If  $P(E)=0.35$ ,  $P(F)=0.43$ .  $P(E|F)=0.52$ :

a) What is  $P(E \cap F)$ ?

b) Are E and F independent?

11. Given this system:

$$2x + 7y + 3z = -30$$

$$3y - 9z = 16$$

$$x + 5y - 3z = -7$$

a) Find the general solution for x, y and z

b) Find a specific solution for when  $x = 3$

12. Glass Creations stained glass company makes windows and lamp shades. A window takes 3 hours of design, 4 hours of cutting and 3 hours of soldering. A lamp shade requires 2 hours of design, 5 hours of cutting and 4 hours of soldering. A window brings in a profit of \$120 and a lampshade gives a profit of \$105. If the company has 15 hours of design available, 25 hours of cutting and 20 hours of soldering per day, how many of each product should they make per day to maximize profit?

13. Jen and Barry's frozen yogurt offers 28 different toppings along with 10 flavors of frozen yogurt.

Their menu has the following options:

Plane Jane: 1 flavor of yogurt and 1 topping

Super Sunday: 3 different flavors of yogurt and 5 different toppings

Topped Off: A bucket of up to 26 different toppings with no yogurt at all

In how many ways can each of these menu items be created?

14. Terry has homework grades of 80, 70, 86, 99, 100 and 100. She also has quiz grades of 80 and 98.

Lastly, on the two midterms she scored 77 and 86. If the syllabus says that the course grade is calculated as: 10% homework, 15% quiz, 25% midterm and 50% final, what is the lowest grade she can get on the final exam and still get at least a B for the course (ie and 80 or higher).

15. A manufacturing plant has a device for testing whether the widgets are working or not. It correctly identifies the working widgets 97% of the time, but misses the broken widgets 7% of the time. On average, one out of every 400 widgets doesn't work. If the machine flags a widget as 'not working', what is the probability that it actually does work?

16. A Car Mechanics class has 15 students who need to be assigned to projects repairing a junk car. 7 need to work on the engine, 4 on the electrical system, 3 on the body and one student needs to be chosen to 'supervise'. In how many ways can the students be assigned jobs?

17. Given the following stochastic matrix: 
$$\begin{matrix} A \\ B \\ C \\ D \end{matrix} \begin{bmatrix} .1 & 0 & .4 & 0 \\ .2 & 1 & 0 & 0 \\ .6 & 0 & .4 & 0 \\ .1 & 0 & .2 & 1 \end{bmatrix}$$
 Do the following:

a) re-write the matrix in standard form  $\begin{bmatrix} I & S \\ 0 & R \end{bmatrix}$

b) Find the fundamental matrix  $(I-R)^{-1}$

c) If you begin in state C, what is the expected number of transitions until being absorbed?

d) Find the stable matrix  $\begin{bmatrix} I & S(I-R)^{-1} \\ 0 & 0 \end{bmatrix}$ . In the long run, what is the probability of starting in state A and eventually winding up in state D?

18. In a nearby park 12% of the trees are suffering from Arboreal Meningitis. A test is administered which accurately detects the disease 99% of the time, but gives a false positive 4% of the time. If you test a tree and the test comes back 'positive', what is the probability that the tree is truly suffering from the dreaded disease?

19. Here's a fun game: You flip a quarter and a dime. If a coin comes up heads then you win 10x the value of the coin, but if it comes up tails you lose 5x the value of the coin. What is a fair price to play this game?

20. You have \$10,000 to invest. You're given an investment opportunity with the following returns and probabilities:

Return	Probability
-\$10,000	0.05
\$0	0.20
\$5,000	0.40
\$20,000	0.35

What is the expected value of the investment, and what is its standard deviation?

21. You have three urns (everybody likes urns). One is red, one is green and one is blue. They contain colored marbles (of course!) The contents are as follows:

- Red urn: 2 red, 4 blue, 4 green marbles
- Blue urn: 7 red, 1 blue, 2 green marbles
- Green urn: 1 red, 1 blue and 8 green marbles

You are incredibly bored one day and decide to start pulling random marbles from the urns. You decide that whatever color you pull out you'll put the marble right back and then move to the urn of the corresponding color for your next pick. You start drawing from the red urn. Approximately what is the probability that the 1,000,000th marble is green?