① $X \sim N(1916.5, 242)$

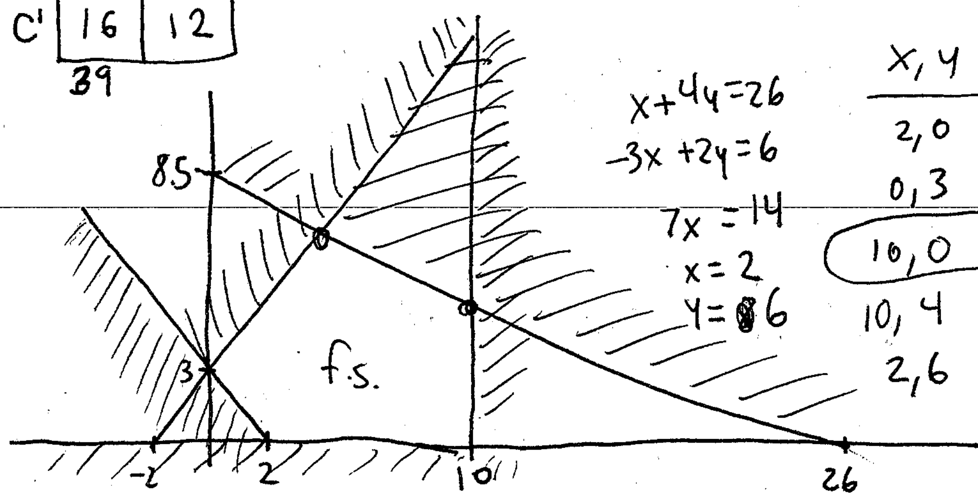
$P(X > 2500) = \text{normalcdf}(2500, 10{,}000, 1916.5, 242) \approx .00795$

②

| | T | T' | |
|---|---|---|---|
| C | 23 | 25 | 48 |
| C' | 16 | 12 | |
| | 39 | | |

76 people surveyed

③




$x + 4y = 26$

$-3x + 2y = 6$

$7x = 14$

$x = 2$

$y = 6$

| $x, y$ | $7x - 2y$ |
|---|---|
| 2, 0 | 14 |
| 0, 3 | -6 |
| 10, 0 | 70 ← maximum |
| 10, 4 | 62 |
| 2, 6 | 2 |

④ $\begin{bmatrix} 5 & 3 \\ -6 & 4 \end{bmatrix}^{-1} = \frac{1}{20+18}\begin{bmatrix} 4 & -3 \\ 6 & 5 \end{bmatrix} = \begin{bmatrix} 4/38 & -3/38 \\ 6/38 & 5/38 \end{bmatrix} = \begin{bmatrix} 2/19 & -3/38 \\ 3/19 & 5/38 \end{bmatrix}$

$\begin{bmatrix} a & a+1 \\ a+2 & a+3 \end{bmatrix}^{-1} = \frac{1}{a(a+3)-(a+1)(a+2)}\begin{bmatrix} a+3 & -a-1 \\ -a-2 & a \end{bmatrix} = \frac{1}{a^2+3a-a^2-3a-2}\left[\ \right]$

$= -\frac{1}{2}\begin{bmatrix} a+3 & -a-1 \\ -a-2 & a \end{bmatrix} = \begin{bmatrix} \frac{-a-3}{2} & \frac{a+1}{2} \\ \frac{a+2}{2} & -a/2 \end{bmatrix}$ all $a$ allowed.

$\begin{bmatrix} 3 & 4 \\ k & 5 \end{bmatrix}^{-1} = \frac{1}{15-4k}\begin{bmatrix} 5 & -4 \\ -k & 3 \end{bmatrix}$ as long as $15 \neq -4k \Rightarrow k \neq -\frac{15}{4}$

⑤ Skip

$X = \#\text{red}$

⑥ $P(\text{at least 10 red socks})$

$= P(X=10) + P(X=11) + \dots + P(X=16)$

$= \dfrac{\binom{46}{10}\binom{63}{6} + \binom{46}{11}\binom{63}{5} + \binom{46}{12}\binom{63}{4} + \binom{46}{13}\binom{63}{3} + \binom{46}{14}\binom{63}{2} + \binom{46}{15}\binom{63}{1} + \binom{46}{16}}{\binom{109}{16}}$

$\approx .0668$

⑦ $A = \begin{array}{c c} & \begin{array}{cc} F & C \end{array} \\ \begin{array}{c} F \\ C \end{array} & \begin{bmatrix} .1 & .8 \\ .9 & .2 \end{bmatrix} \end{array}$ $P_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P_4 = A^4 P_0 = \begin{bmatrix} .3576 \\ .6424 \end{bmatrix}$

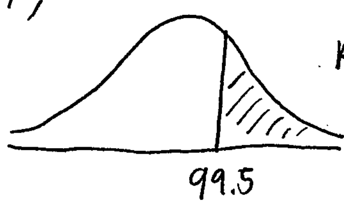
$\begin{bmatrix} .1 & .8 \\ .9 & .2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

$.1x + .8y = x$

$-.9x + .8y = 0$

Solve $\begin{cases} x + y = 1 \\ -.9x + .8y = 0 \end{cases} \rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8/17 \\ 9/17 \end{bmatrix}$

⑧ $p = \frac{2}{7}$, $n = 230 \rightarrow \mu \approx 65.714$ $\sigma \approx 6.8512$

a) 


~~Normal~~ $P(Y \geq 99.5) \approx$ normalcdf$(99.5, 230, 65.714, 6.8512)$

$\approx 0$

b) $P(79.5 \leq Y \leq 91.5) =$ normalcdf$(79.5, 91.5, 65.714, 6.8512)$

$\approx .022$ or $2.2\%$

⑨ $A = \begin{array}{c c} & \begin{array}{cc} S & A \end{array} \\ \begin{array}{c} S \\ A \end{array} & \begin{bmatrix} .2 & .5 \\ 0 & .1 \end{bmatrix} \end{array}$ $D = \begin{bmatrix} 500 \\ 1000 \end{bmatrix}$

$X - AX = D$

$(I - A)X = D$

$X = (I-A)^{-1}D = \begin{bmatrix} 1319.44 \\ 1111.11 \end{bmatrix}$

Must produce \$1,319.44 suffering and \$1,111.11 art.

⑩

| | E | E' | |
|---|---|---|---|
| F | .1436 | .4264 | .57 |
| F' | .2064 | .2236 | .43 |
| | .35 | .65 | |

$P(E' \mid F') = \dfrac{P(E' \cap F')}{P(F')} \Rightarrow .52 = \dfrac{P(E' \cap F')}{.43}$

$P(E' \cap F') = .2236$

a) $P(E \cap F) = .1436$

b) $(.57)(.35) = .1995 \neq .1436$

so E and F are not independent.

(11) $\text{rref} \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 12 & -101/3 \\ 0 & 1 & -3 & 16/3 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $\quad x + 12z = \frac{-101}{3}$ , $y \cancel{+} -3z = \frac{16}{3}$ $\rightarrow$ $\boxed{x = \frac{-101}{3} - 12z,\quad y = 3z + \frac{16}{3},\quad z = \text{any real}}$

$$3 = \frac{-101}{3} - 12z$$

$$9 = -101 - 36z$$

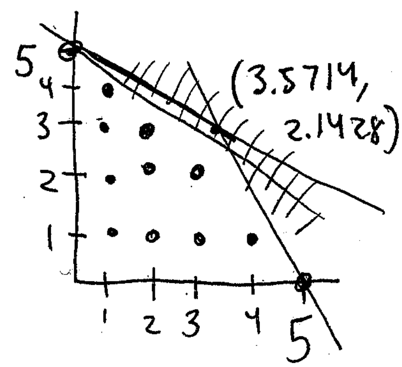
$$110 = -36z \quad \text{so } z = \frac{-101}{36}, \quad y = 3\left(\frac{-101}{36}\right) + \frac{16}{3} = \frac{-37}{12}$$

$$\boxed{x = 3,\ y = -\frac{37}{12},\ z = \frac{-101}{36}}$$

(12)

| | window | shade | avalable |
|---|---|---|---|
| design | 3 | 2 | 15 |
| cutting | 4 | 5 | 25 |
| soldering | 3 | 4 | 20 |
| profit | \$120 | \$105 | |

$$3x + 2y \le 15$$
$$4x + 5y \le 25$$
$$3x + 4y \le 20$$
$$x \ge 0,\ y \ge 0$$




| maximize | $120x + 105y$ |
|---|---|
| 0, 0 | |
| 0, 5 | 525 |
| 5, 0 | 600 |
| 3.5714, 2.1428 | 653.56 |

but can't make part of a lampshade. try $(3,2) \rightarrow 570$

not as good as $(5,0)$.

Solution: Just make windows.

13) a) Plane Jane 280 types

b) Super Sunday $\binom{10}{3}\binom{28}{5} = 11,793,600$

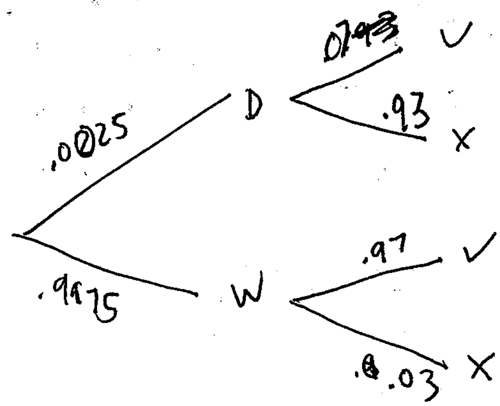
c) $2^{28} - \binom{28}{28} - \binom{28}{27} = 2^{28} - 29$

14) $.10\left(\frac{80+70+86+99+100+100}{6}\right) + .15\left(\frac{80+98}{2}\right) + .25\left(\frac{77+86}{2}\right) + .5x \ge 80$

$42.6417 + .5x \ge 80$

$.5x \ge 37.3583$

$x \ge 74.71667$

15) 


$P(w|v) = \frac{(.97)(.9975)}{(.97)(.9975) + (.07)(.0025)}$

$= .9998$

16) $\frac{15!}{7!\,4!\,3!} = 1,801,800$

17) a)
$$\begin{array}{c} B \\ D \\ A \\ C \end{array} \begin{bmatrix} 1 & 0 & .2 & 0 \\ 0 & 1 & .1 & .2 \\ 0 & 0 & .1 & .4 \\ 0 & 0 & .6 & .4 \end{bmatrix}$$

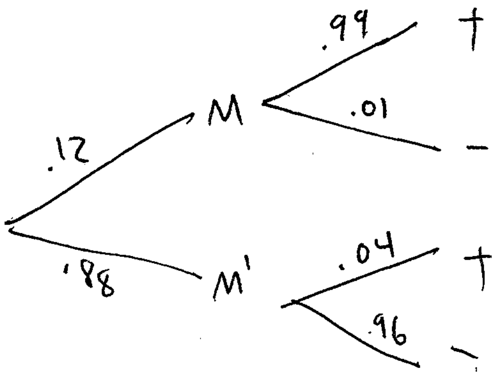
b) $(I-R)^{-1} = \begin{bmatrix} 2 & 4/3 \\ 2 & 3 \end{bmatrix}$

c) $\frac{13}{3}$ transitions

d)
$$\begin{array}{c} B \\ D \\ A \\ C \end{array} \begin{bmatrix} 1 & 0 & .4 & 4/15 \\ 0 & 1 & .6 & 11/15 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

.6

(18)




$$P(M|+) = \frac{(.12)(.99)}{(.12)(.99) + (.88)(.04)}$$

$$\approx .7714$$

(19)

| outcome | \$ | prob |
|---|---|---|
| $H_D H_Q$ | 3.50 | 1/4 |
| $H_D T_Q$ | -.25 | 1/4 |
| $T_D H_Q$ | 2.00 | 1/4 |
| $T_D T_Q$ | -1.75 | 1/4 |

Fair Price is \$.875

(20) $E(X) = \$8,500$

$\sigma_X = \$9,096.70$

(21)

$$A = \begin{array}{c c} & \begin{array}{ccc} R & B & G \end{array} \\ \begin{array}{c} R \\ B \\ G \end{array} & \begin{bmatrix} .2 & .7 & .1 \\ .4 & .1 & .1 \\ .4 & .2 & .8 \end{bmatrix} \end{array} \longrightarrow A^{100} = \begin{bmatrix} 2/9 & 2/9 & 2/9 \\ 1/6 & 1/6 & 1/6 \\ 11/18 & 11/18 & 11/18 \end{bmatrix}$$

Probability of 11/18 that the millionth marble is green.