2.3 Matrix Operations

There are three operations we must define for working with matrices: Addition, Scalar Multiplication and Matrix Multiplication.

**Matrix Addition**
Given two Matrices A and B, both of which are of dimensions nxm, we have A+B=C, where C is also of dimensions nxm, and each entry of matrix C is $c_{i,j}=a_{i,j}+b_{i,j}$.

ex) \[
\begin{bmatrix}
4 & 5 & 1 \\
2 & 4 & -1
\end{bmatrix} +
\begin{bmatrix}
-1 & 4 & 2 \\
1 & 3 & -3
\end{bmatrix} =
\begin{bmatrix}
4-1 & 5+4 & 1+2 \\
2+1 & 4+3 & -1-3
\end{bmatrix} =
\begin{bmatrix}
3 & 9 & 3 \\
3 & 7 & -4
\end{bmatrix}
\]

ex) \[
\begin{bmatrix}
3 & 8 & 9 \\
3 & 4 & 5
\end{bmatrix} +
\begin{bmatrix}
4 & 7 \\
0 & 1 \\
4 & -1
\end{bmatrix}
\]

is not defined because the two matrices have different dimensions.

**Scalar Multiplication**
Given a matrix A and a real number c, $cA=B$ where each entry of B is $b_{i,j}=ca_{i,j}$. In other words, we multiply each entry of matrix A by the number c.

ex) \[
3 \begin{bmatrix}
4 & 5 & 1 \\
2 & 4 & -1
\end{bmatrix} =
\begin{bmatrix}
3(4) & 3(5) & 3(1) \\
3(2) & 3(4) & 3(-1)
\end{bmatrix} =
\begin{bmatrix}
12 & 15 & 3 \\
6 & 12 & -3
\end{bmatrix}
\]

**Matrix Multiplication**
Given a matrix A with dimensions nxm and B which is mxp, $AB=C$, where matrix C has dimensions nxp, and each entry of matrix C is $c_{i,j}=\sum a_{i,k}b_{k,j}$.
In other words, for the entry in row i and column j of the product matrix, we take row i of matrix A and column j of matrix B, and take the sum of the products of each corresponding entry.

ex) \[
\begin{bmatrix}
3 & 8 & 9 \\
3 & 4 & 5
\end{bmatrix} \begin{bmatrix}
4 & 7 \\
0 & 1 \\
4 & -1
\end{bmatrix}
\]

First of all, because we are multiplying a [2x3] and [3x2] matrix, this is allowed. The columns of the first must be the same as the rows of the second. The product will be a 2x2 matrix (the rows of the first and columns of the second).

Entry 1,1 of the product comes from row 1 of the first matrix and column 1 of the second matrix
\[
\begin{bmatrix}
3 & 8 & 9 \\
3 & 4 & 5
\end{bmatrix} \begin{bmatrix}
4 & 7 \\
0 & 1 \\
4 & -1
\end{bmatrix} =
\begin{bmatrix}
3(4)+8(0)+9(4) & ? \\
3(4)+4(0)+5(4) & ?
\end{bmatrix}
\]

The remaining entries are calculated accordingly
\[
\begin{bmatrix}
48 & 3(7)+8(1)+9(-1) \\
3(4)+4(0)+5(4) & 3(7)+4(1)+5(-1)
\end{bmatrix} =
\begin{bmatrix}
48 & 20 \\
22 & 20
\end{bmatrix}
\]

ex) \[
\begin{bmatrix}
1 & 8 & -2 \\
4 & 7 & 9 \\
-2 & 2 & 4
\end{bmatrix} \begin{bmatrix}
2 & 4 & 3 \\
-1 & 2 & 4
\end{bmatrix}
\]

is not defined because the matrices are 3x3 and 2x3.