**Math 160 Discussion Notes**
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## 2.3 Matrix Operations

There are three operations we must define for working with matrices: Addition, Scalar Multiplication and Matrix Multiplication.

### Matrix Addition

Given two Matrices A and B, both of which are of dimensions nxm, we have A+B=C, where C is also of dimensions nxm, and each entry of matrix C is $c_{i,j}=a_{i,j}+b_{i,j}$.

ex) $\begin{bmatrix} 4 & 5 & 1 \\ 2 & 4 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 & 2 \\ 1 & 3 & -3 \end{bmatrix} = \begin{bmatrix} 4-1 & 5+4 & 1+2 \\ 2+1 & 4+3 & -1-3 \end{bmatrix} = \begin{bmatrix} 3 & 9 & 3 \\ 3 & 7 & -4 \end{bmatrix}$

ex) $\begin{bmatrix} 3 & 8 & 9 \\ 3 & 4 & 5 \end{bmatrix} + \begin{bmatrix} 4 & 7 \\ 0 & 1 \\ 4 & -1 \end{bmatrix}$ is not defined because the two matrices have different dimensions.

### Scalar Multiplication

Given a matrix A and a real number c, cA=B where each entry of B is $b_{i,j}=c*a_{i,j}$. In other words, we multiply each entry of matrix A by the number c.

ex) $3\begin{bmatrix} 4 & 5 & 1 \\ 2 & 4 & -1 \end{bmatrix} = \begin{bmatrix} 3(4) & 3(5) & 3(1) \\ 3(2) & 3(4) & 3(-1) \end{bmatrix} = \begin{bmatrix} 12 & 15 & 3 \\ 6 & 12 & -3 \end{bmatrix}$

### Matrix Multiplication

Given a matrix A with dimensions nxm and B which is mxp, AB=C, where matrix C has dimensions nxp, and each entry of matrix C is $c_{i,j}=a_{i,1}b_{1,j}+a_{i,2}b_{2,j} + \ldots + a_{i,m}b_{m,j}$

In other words, for the entry in row i and column j of the product matrix, we take row i of matrix A and column j of matrix B, and take the sum of the products of each corresponding entry.

ex) $\begin{bmatrix} 3 & 8 & 9 \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 4 & 7 \\ 0 & 1 \\ 4 & -1 \end{bmatrix}$

First of all, because we are multiplying a [2x3] and [3x2] matrix, this is allowed. The columns of the first must be the same as the rows of the second. The product will be a 2x2 matrix (the rows of the first and columns of the second).

Entry 1,1 of the product comes from row 1 of the first matrix and column 1 of the second matrix

$$\begin{bmatrix} \mathbf{3} & \mathbf{8} & \mathbf{9} \\ 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} \mathbf{4} & 7 \\ \mathbf{0} & 1 \\ \mathbf{4} & -1 \end{bmatrix} = \begin{bmatrix} 3(4)+8(0)+9(4) & ? \\ ? & ? \end{bmatrix}$$

The remaining entries are calculated accordingly

$$= \begin{bmatrix} 48 & 3(7)+8(1)+9(-1) \\ 3(4)+4(0)+5(4) & 3(7)+4(1)+5(-1) \end{bmatrix} = \begin{bmatrix} 48 & 20 \\ 22 & 20 \end{bmatrix}$$

ex) $\begin{bmatrix} 1 & 8 & -2 \\ 4 & 7 & 9 \\ -2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ -1 & 2 & 4 \end{bmatrix}$ is not defined because the matrices are 3x3 and 2x3.