Math 160, Finite Mathematics for Business

Section 6.2: Intro to Probability - Discussion Notes
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The foundation of probability is Set Theory. In order to work with probabilities, you need to understand sets - intersections, unions, complements. Special attention should be made to the following terms, which are important to the discussions of probability:
experiment: An activity which has some observable result
outcome: One particular result from an experiment
sample space: The set containing every outcome of an experiment; the sample space is usually called S.
event: A subset of the sample space; an event describes some set of outcomes.
An event that corresponds to the empty set $\emptyset$ is called an **impossible event**.
An event that corresponds to the entire sample space S is called a **certain event**.

6.2.3) A letter is chosen at random from the word MISSISSIPPI.
a) What is the sample space?
An outcome of the experiment is a letter from the word, and because the first S and the second S are indistinguishable, the set of all outcomes will not have repeats in it. So The sample space is $\{M, I, S, P\}$.
b) Describe the event "The letter chosen is a vowel" in terms of the sample space. The only outcome from the sample space which corresponds to this event is choosing the letter "I", and because an event must be represented as a subset of the sample space, this event is $\{I\}$.

6.2.4) Two 4-sided dice are rolled (yes, 4-sided dice exist - just ask any D&D player). Assume the dice are distinguishable from each other.
a) What is the sample space?
Because the dice are distinguishable (for example, a white die and a red die) we represent the outcomes as ordered pairs of numbers. So the entire sample space consists of all possible pairs of numbers:
$\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4)\}$
b)describe the subsets (i) Both numbers are even, (ii) The sum is 6, (iii) 2 or three occurs but the two numbers are not the same.
(i) The event is a subset of our sample space, consisting of $\{(2,2),(2,4),(4,2),(4,4)\}$.
(ii) The event is $\{(2,4),(3,3),(4,2)\}$.
(iii) $\{(1,2),(1,3),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,2),(4,3)\}$

6.2.5) You have two urns, Urn I and Urn II. Each is filled with both red and white marbles. You choose an urn and remove one marble, noting its color. What is the sample space?
Each outcome is described by the urn in question and the color of the marble. Let's use R and W for red and white. So the set of all possible outcomes is: $\{(I,R),(I,W),(II,R),(II,W)\}$.

Because events are sets, we can consider the union, intersection and complement of events. If we have two generic events E and F:
The union E∪F means "Either E or F occurs (or both)"
The intersection E∩F means "Both E and F occur"
The complement E' means "E does not occur"
Two events E and F are said to be **mutually exclusive** when their intersection is the empty set (E∩F= $\emptyset$); i.e. the two events share no outcomes in common.

6.2.10) You go to the parking lot and pick a car at random, noting its color and make. Consider the following events:
E: "the car is red"
F: "the car is a Chevrolet"

G: "the car is a green Ford"
H: "the car is black or a Chrysler"
Describe the following events in words:
E∩F: The car is a red Chevrolet
G': The car is not a green Ford
E∪G: The car is red OR a green Ford
G∩H: The car is a black green Ford or the car is a green Ford Chrysler. (Assuming a car can only have 1 color and 1 make, this event is nonsensical, and describes an empty set)
E'∩F' The car is not red and the car is not a Chevrolet. (note that this is not the same as "the car is not a red Chevrolet". For example, a green Chevrolet would not be in the former set, but it would be allowed into the latter.

Which events are mutually exclusive?
F and G: Yes, these are mutually exclusive - the car cannot be a Chevrolet and a green Ford.
E' and G: No, these are not mutually exclusive; a car can be "not red" and a green Ford at the same time.
E and H: No, these are not mutually exclusive. A red Chrysler falls into both sets.
F' and H': Not mutually exclusive.

6.2.17) Toss 10 coins at the same time and observe and record the number of heads. What is the sample space?
In this experiment the result being observed is not "H" or a sequence of ten Hs and Ts. What we are observing is the **number of heads**. Some possible outcomes would be 3, 6, 10 or 0. Any number from 0 to 10 is a possible outcome, so our sample space is $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
describe the event "there are more heads than tails" as a subset of the sample space.
Because there are 10 coins, in order for more to be heads than tails, there would have to be 6 or more heads. Thus the event is the subset $\{6, 7, 8, 9, 10\}$.

6.2.23) Balls numbered #1-#9 are in a jar. You draw one, replace it and then draw another one. Give an example of one outcome.
There are many examples. One would be that the first ball is numbered 4 and the second draw is ball number 8. We can use an ordered pair to denote this outcome as (4,8).
How many outcomes are in the sample space?
Because there are 9 possibilities for the first ball and 9 for the second (the first ball drawn is put back into the jar) there will be 81 outcomes in the sample space.