Math 160, Finite Mathematics for Business

Section 6.4: Calculating Probabilities of Events - Discussion Notes
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If a sample space has N equally likely outcomes and we want to calculate the probability of event E, we have:

$$Pr(E) = \frac{\#\,events\,in\,E}{N}$$

We will rely heavily on the techniques developed in chapter 5 to count the number of outcomes in events in order to calculate probabilities.

Complement Rule: Because the number of events in E' is [# events in S] - [# events in E] it can be shown that $Pr(E') = 1 - Pr(E)$, and $Pr(E) = 1 - Pr(E')$. This rule can come in very handy because sometimes it is faster to count the number of events in E' than E.

6.4.2) Michael and Christopher are in a 7 person race around a track. The track has 7 lanes and the runners will be randomly assigned a lane. What is the probability that Michael is assigned the inside lane and Christopher is assigned the outside lane?
We need to calculate the number of outcomes in this event. Of the seven runners, Michael must be in the inside lane and Christopher must be in the outside lane, but the other 5 runners could be arranged in any way in the 5 remaining lanes. This is a permutation of 5 out of 5 people: 5!. The number of outcomes in the sample space (the number of ways all 7 runners could be arranged) is 7!. So our answer is $\frac{5!}{7!} = \frac{5!}{7 \cdot 6 \cdot 5!} = \frac{1}{42}$.

6.4.6) An Urn contains 40 balls, each is either red or white. If the probability of selecting a red ball is .45, how many red balls are in the urn?
The probability of selecting a red ball is [# red balls in the urn] divided by [# balls in the urn]. We can assign the variable r=# red balls. So $Pr(Red) = \frac{r}{\#\,balls\,in\,the\,urn} \rightarrow .45 = \frac{r}{40}$. We multiply both sides by 40 to get $18 = r$.

6.4.11) A classroom has 12 boys and 10 girls. 7 students are randomly chosen to go up to the board and work on problems.
a) What is the probability that there are at least two girls going up to the board?
We want to count the number of outcomes in this event, which is the sum of the cases 2 girls, 3 girls, ..., 7 girls. We could find these six numbers but it is easier to employ the complement rule.
$Pr(at\,least\,2\,girls) = 1 - Pr(1\,or\,0\,girls)$
The number of ways of choosing 1 girl really means [# ways of choosing 1 girl and 6 boys], because we need to account for the ways of picking all 7 of the children. This is $\binom{10}{1}\binom{12}{6}$. The number of ways of choosing 0 girls is $\binom{10}{0}\binom{12}{7} = \binom{12}{7}$. For the denominator we need to count the number of ways of choosing 7 kids from the 22 in class - this is simply $\binom{22}{7}$. So our answer is:
$Pr(at\,least\,2\,girls) = 1 - (Pr(1\,girl) + Pr(0\,girls)) = 1 - (\frac{\binom{12}{6}\binom{10}{1}}{\binom{22}{7}} + \frac{\binom{12}{7}}{\binom{22}{7}}) \approx .94$

b) What is the probability that more boys than girls go up to the board?
This event describes the union of events "4 boys and 3 girls", "5 boys and 2 girls", "6 boys and 1 girl" and "7 boys". The numbers of outcomes in each case are $\binom{12}{4}\binom{10}{3} = 59,400$, $\binom{12}{5}\binom{10}{2} = 35,640$, $\binom{12}{6}\binom{10}{1} = 9,240$ and $\binom{12}{7} = 792$ respectively, and the number of ways that 7 kids can be chosen as calculated in part a) is $\binom{22}{7} = 170,544$. Thus, our probability is calculated:
$\frac{59{,}400+35{,}640+9{,}240+792}{170{,}544} \approx .62$.

c) What is the probability that no boys are chosen to go up to the board?
The number of ways no boys can be chosen is $\binom{10}{7} = 120$. Our probability is calculated $\frac{120}{170{,}544} \approx .0007$.

d) If the sudents are chosen one by one, what is the probability that the first three students are all boys?

This question changes things significantly. We are only looking at ways of choosing the first 3 students, and because they are done in order it is actually calculated using a permutation, rather than a combination. $Pr(first\ 3\ are\ boys) = \frac{Permutations\ of\ 3\ boys\ out\ of\ 12}{Permutations\ of\ 3\ students\ out\ of\ 22} = \frac{12 \cdot 11 \cdot 10}{22 \cdot 21 \cdot 20} = \frac{2}{7}$.

6.4.35) What is the probability that a 5 senator committee in congress (which is randomly appointed) consists of senators representing 5 different states?

In chapter 5 we have calculated the number of 5 senator committees representing 5 states as $\binom{50}{5} \cdot 2^5 = 67,800,320$. This will be in the numerator of our probability. The denominator is the number of 5 senator committees. Because we are choosing 5 senators from the 100 in the senate, this is $\binom{100}{5} = 75,287,520$. Our probability is thus $\frac{67,800,320}{75,287,520} \approx .90$.

6.4.38) What is the probability that a random arrangement of the letters in the word GEESE is a word with all three Es adjacent?

We can use the ordered partition method to calculate the denominator of the probability (the number of distinct words that can be made by rearranging the 5 letters). This is $\frac{5!}{3!} = 20$. We divide by 3! because there is one letter repeated 3 times. See the example in the Quiz 4 review for more information.

To count the number of words with 3 Es adjacent we could count arrangements of "G", "S" and "EEE". There are $3! = 6$ permutations of these three things, and that counts up all the words with all 3 Es adjacent. Our probability is $\frac{6}{20} = \frac{3}{10}$.

If you are dealt a 5 card poker hand, what is the probability of getting a full house (Three of a kind and a pair)?

We have previously calculated that there are $\binom{52}{5} = 2,598,960$ possible 5 card poker hands, so this goes in our denominator. To count the number of Full Houses, we use the method developed in chapter 5:

\# Possibilities of the card value for the triple: $\binom{13}{1}$

\# possibilities of suits in our triple: $\binom{4}{3}$

\# possibilities of the card value for the pair: $\binom{12}{1}$ (12 because the value in our triple is not a possibility)

\# possibilities of suits for the pair: $\binom{4}{2}$

The product of all four numbers is: $\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2} = 3,744$.

The probability of a full house is $\frac{3,744}{2,598,960} \approx 0.0014$.