Math 160, Finite Mathematics for Business

Section 6.5: Conditional Probability and Independence - Discussion Notes
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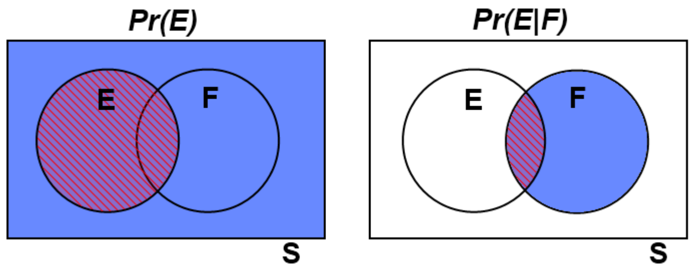
Conditional Probability: The conditional probability is the probability of one event, E, happening with the prior knowledge that another event F has occured. In practice, we say "the probability of E given F" and write $Pr(E|F)$. The formula is as follows:

$$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)}$$

Perhaps it is helpful to think of this using Venn Diagrams. The normal probability of event E is the number of events in E over the number of events in S, the sample space. When we look at the conditional probability it is as if we are constricting th sample space to only the set F - this is illustrated below:



Another formula for calculating conditional probability is given as follows:

$$Pr(E|F) = \frac{\#\ of\ outcomes\ in\ E \cap F}{\#\ of\ outcomes\ in\ F}$$

It should be noted that conditional probability can only be calculated if $Pr(F) \neq 0$.

Independence: If two events are independent, then the occurence of one will not affect the occurence of the other. Common examples would be the probability of rolling a 5 on the first roll of a die and a 3 on the second roll of a die. We have three equivalent definitions of indepenence, and if any one of them is true then all three are true:

$Pr(E \cap F) = Pr(E) \cdot Pr(F)$
$Pr(E|F) = Pr(E)$
$Pr(F|E) = Pr(F)$

If you have more than 2 events (to be general, n events $E_1, E_2, ..., E_n$), then they are independent if:

$$Pr(E_1 \cap E_2 \cap \cdots \cap E_n) = Pr(E_1) \cdot Pr(E_2) \cdots Pr(E_n)$$

6.5.5) In a certain town there is a .001 probability of cancer among the residents. Also 30% of the residents work for the Ajax company in town. It is found that among Ajax employees, the rate of cancer is .001. Are having cancer and working for Ajax independent?
Let C:"has cancer" and A:"works for Ajax". The problem gives us the following probabilities:
$Pr(C) = .001$
$Pr(A) = .30$
$Pr(C|A) = .003$
Notice the last one is not $Pr(C \cap A)$. $C \cap A$, which would be "Probability that someone works for Ajax AND has cancer". $Pr(C|A)$ is "Probability someone has cancer GIVEN THAT he works for Ajax". Note that this is different than $Pr(A|C)$ which is "Probability someone works for Ajax GIVEN THAT he has cancer".
Anyhow, because $Pr(C|A) \neq Pr(C)$, we can say that A and C are not independent.

6.5.12) If you have events E and F such that: $Pr(E) = .3, Pr(F) = .6$**, and** $Pr(E \cup F) = .7$**,**
a) What is $Pr(E \cap F)$**?**
By the Inclusion-Exclusion formula, $Pr(E \cap F) = Pr(E) + Pr(F) - Pr(E \cup F) = .3 + .6 - .7 = .2$

b) What is $Pr(E|F)$?
$Pr(E|F) = \frac{Pr(E \cap F)}{Pr(F)} = \frac{.2}{.6} = \frac{1}{3}$

c) What is $Pr(F|E)$?
Note first that E∩F = F∩E, so $Pr(F|E) = \frac{Pr(F \cap E)}{Pr(E)} = \frac{.2}{.3} = \frac{2}{3}$

d) What is $Pr(E' \cap F)$?
We have Pr(F)=.6 and Pr(E∩F)=.2. This means Probability of E and F is .2, so what is probability of (not E) and F? This is $.6 - .2 = .4$. If the probability of the set F is .6 and the portion that intersects E is .2, the rest of F must be in the intersection of E'.

e) What is $Pr(E'|F)$?
By the formula, $Pr(E'|F) = \frac{Pr(E' \cap F)}{Pr(F)} = \frac{.4}{.6} = \frac{2}{3}$

f) Are E and F independent?
No, because $Pr(E) = .3$ and $Pr(E|F) = \frac{1}{3} \approx .33333$, which are not equal.

6.5.43) There are 25 balls in an urn: 10 red and 15 white. If the balls are sampled without replacement, which is more likely: pulling a red ball on the first try, or bulling a red ball on the second?
Let R: Red, and W: White
Probability of Red on the first try is $\frac{10}{25} = .40$
Probability of red on the second depends on what you get on the first draw. We have to look at two cases: Pr(RR) and Pr(WR), meaning the probability of red then red, and white then red.
ways to draw two reds would be $P(10, 2) = 10 \cdot 9 = 90$
ways to draw white then red would be $15 \cdot 10 = 150$
ways to draw 2 balls is $P(25, 2) = 25 \cdot 24 = 600$
So our probability is:
$Pr(second\, ball\ R) = \frac{90}{600} + \frac{150}{600} = \frac{90+150}{600} = .40$
So it turns out that these two events are equally likely. (This isn't ALWAYS the case, so don't assume this result applies whenver you have a similar problem.)