Math 160, Finite Mathematics for Business

Section 6.7: Bayes' Theorem - Discussion Notes
Brian Powers - TA - Fall 2011

Bayes' Theorem allows us to calculate conditional probabilities without drawing an entire Tree. If we look at a general case where we have events B_1 and B_2 that are mutually exclusive (i.e. $B_1 \cap B_2 = \emptyset$) and cover the entire sample space (i.e. $B_1 \cup B_2 = S$), and we want to consider some other event A in the sample space, Bayes' Theorem says:

$$Pr(B_1|A) = \frac{Pr(B_1) \cdot Pr(A|B_1)}{Pr(B_1) \cdot Pr(A|B_1) + Pr(B_2) \cdot Pr(A|B_2)}$$

If the sample space is partitioned into B_1, B_2, and B_3, then

$$Pr(B_1|A) = \frac{Pr(B_1) \cdot Pr(A|B_1)}{Pr(B_1) \cdot Pr(A|B_1) + Pr(B_2) \cdot Pr(A|B_2) + Pr(B_3) \cdot Pr(A|B_3)}$$

6.7.2) A company manufactures electronics using 6 types of transistors, each with its own failure rate. The chart below gives the proportion of each type of transistor in the device along with its corresponding failure rate:

Type	**Proportion**	**Failure Rate**
1	**.30**	**.0002**
2	**.25**	**.0004**
3	**.20**	**.0005**
4	**.10**	**.001**
5	**.05**	**.002**
6	**.10**	**.004**

If a transistor has failed, what is the probability it is Type 1?
In this problem we can re-label the columns to more easily plug numbers into Bayes' theorem. If the Type=n, Proportion is actually $Pr(Type\, n)$, and Failure Rate is $Pr(Fail|Type\, n)$, that is, probability of failure given type=n. We can add another column: $Pr(Type\, n) \cdot Pr(Fail|Type\, n)$.

Type n	$Pr(Type\, n)$	$Pr(Fail\|Type\, n)$	$Pr(Type\, n) \cdot Pr(Fail\|Type\, n)$
1	.30	.0002	.00006
2	.25	.0004	.0001
3	.20	.0005	.0001
4	.10	.001	.0001
5	.05	.002	.0001
6	.10	.004	.0004

By Bayes' Theorem, $Pr(Type = 1|Fail) = \frac{.00006}{.00006+.0001+.0001+.0001+.0001+.0004} = \frac{.00006}{.00086} \approx .06984$

6.7.6) People are taking a survey at the exit polls in a small town. The following table gives the proportion of registered voters and their respective turnout percentages:

Party	**% Registered**	**% Turnout**
Democrat	**50**	**40**
Republican	**20**	**50**
Independent	**30**	**70**

If a random voter is questioned exiting the voting booth, what is the probability he or she will be registered Independent?
The question is really asking for the probability a voter is Independent GIVEN THAT he or she turned out for the vote. This conditional probability can be calculated using Bayes' Theorem. To simplify notation, let events D: "Is a Democrat", R: "Is a Republican", I: "Is an Independent" and T: "Turned out to vote".
$Pr(I|T) = \frac{Pr(I)Pr(T|I)}{Pr(D)Pr(T|D)+Pr(R)Pr(T|R)+Pr(I)Pr(T|I)} = \frac{.30 \cdot .70}{.50 \cdot .40+.20 \cdot .50+.30 \cdot .70} \approx .4118$

6.7.12) A company is having random drug tests in the workplace. The lab produces false negatives 2% of the time and false positives 5% of the time. 10% of the employees at this company use drugs.
a) If an employee tests positive, what is the probability that he uses drugs?
Let's use D: "uses drugs", D': "does not use drugs", and P: "tests positive".
$Pr(D) = .10$
$Pr(D') = .90$
$Pr(P|D) = .98$ (2% False negatives means for drug users, 2% will get a Negative result, so 98% will get POS)
$Pr(P|D') = .05$ (This is what 5% false positives means)
Bayes' Theorem gives us:
$Pr(D|P) = \frac{Pr(D)Pr(P|D)}{Pr(D)Pr(P|D)+Pr(D')Pr(P|D')} = \frac{.10 \cdot .98}{.10 \cdot .98+.90 \cdot .05} \approx .6853$
Surprisingly, a positive result on the drug test only means there is a 69% chance the employee is actually using drugs.

What is the probability a non-drug user tests positive twice in a row?
Because there is a 5% chance a non-drug user tests positive once, the probability that it happens twice is $.05 \cdot .05 = .0025$

What is the probability that someone who tests positive twice in a row is not a drug user?
Let's use P×2: "tests positive twice". By Bayes' Theorem,
$Pr(D'|P \times 2) = \frac{Pr(D')Pr(P \times 2|D')}{Pr(D')Pr(P \times 2|D')+Pr(D)Pr(P \times 2|D)}$
We have $Pr(P \times 2|D') = .0025$ from part b), but we need to calculate $Pr(P \times 2|D)$. This is $Pr(P|D) \cdot Pr(P|D) = .98 \cdot .98$ or $.98^2$. So our probability is:
$\frac{.90 \cdot .0025}{.90 \cdot .0025+.10 \cdot .98^2} \approx .0229$