**Math 160, Finite Mathematics for Business**
Section 7.2: Frequency and Probability Distributions – Discussion Notes
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Recall that the **frequency** is the number of observations of a specific outcome. The **relative frequency** is a proportion of all observations (frequency / total observations)

A **frequency distribution** or **relative frequency distribution** is a way of displaying observed samples from experiments, whereas a probability distribution is a theoretical model for an experiment.

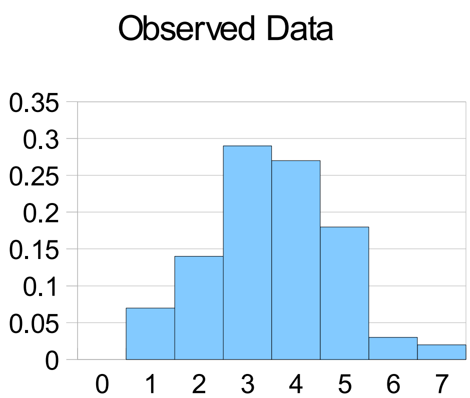
For example, let's say we have 100 students flip a penny 7 times, each one records the number of heads observed. The observations are put in the following table:

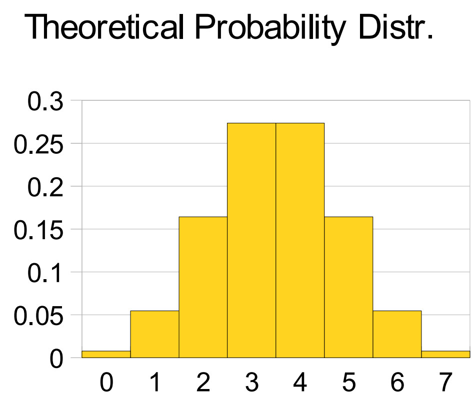
| # Heads | Frequency | Relative Frequency |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 7 | 0.07 |
| 2 | 14 | 0.14 |
| 3 | 29 | 0.29 |
| 4 | 27 | 0.27 |
| 5 | 18 | 0.18 |
| 6 | 3 | 0.03 |
| 7 | 2 | 0.02 |

Incidentally, the theoretical probability distribution for this experiment (we will see how this is constructed in section 7.3) is:

| # Heads | Probability |
|---|---|
| 0 | 0.0078 |
| 1 | 0.0547 |
| 2 | 0.1641 |
| 3 | 0.2734 |
| 4 | 0.2734 |
| 5 | 0.1641 |
| 6 | 0.0547 |
| 7 | 0.0078 |

We can create a histogram for our relative frequency distribution as follows:




You can see that the two histograms are similar.

**ex) An urn contains 3 red balls and 4 white balls. You sample 3 balls and observe the number of red balls. Make a histogram for the probability distribution.**

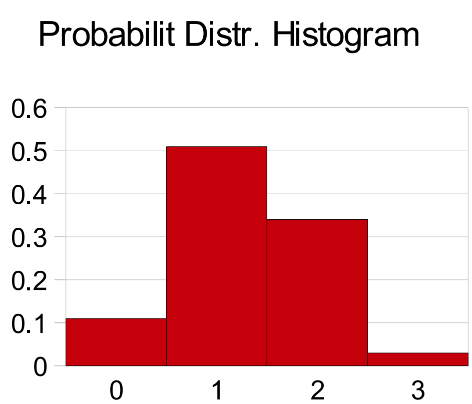
Let X be the random variable representing the number of red balls observed after sampling from the urn. Because there are 3 reds in the urn and we are sampling three at a time, it is possible that the number of red balls can be 0, 1, 2 or 3.

The number of possible outcomes is $\binom{7}{3}=35$

The number of ways of getting 0 red = # ways of getting 0 red and 3 white = $\binom{3}{0}\cdot\binom{4}{3}=4$

The probability distribution and histograms can be created as follows:

| k | Pr(X=k) |
|---|---|
| 0 | $\frac{\binom{3}{0}\cdot\binom{4}{3}}{35}=\frac{4}{35}\approx .11$ |
| 1 | $\frac{\binom{3}{1}\cdot\binom{4}{2}}{35}=\frac{18}{35}\approx .51$ |
| 2 | $\frac{\binom{3}{2}\cdot\binom{4}{1}}{35}=\frac{12}{35}\approx .34$ |
| 3 | $\frac{\binom{3}{3}\cdot\binom{4}{0}}{35}=\frac{1}{35}\approx .03$ |



**ex) Given the following probability distributions, answer the questions:**

| **k** | **Pr(X=k)** | **Pr(Y=k)** |
|---|---|---|
| **1** | **0.3** | **0.2** |
| **2** | **0.4** | **0.2** |
| **3** | **0.2** | **0.2** |
| **4** | **0.1** | **0.4** |

**a) Pr(X=2 or 3)** = Pr(X=2)+Pr(X=3) because these are disjoint events
= 0.4+0.2 = 0.6

**b) Pr(Y=2 or 3)** = Pr(Y=2)+Pr(Y=3) because these are disjoint
= 0.2+0.2=0.4

**c) Pr(X ≥ 2)** = Pr(X=2 or X=3 or X=4) = 0.4+0.2+0.1=0.7

**d) Probability that X+3 is at least 5**
= Pr(X+3 ≥ 5)
= Pr(X ≥ 2) by subtracting 3 from both sides of the inequality
= 0.7 from part c

**e) Probability that $Y^2$ is at most 9**
= Pr($Y^2 \leq 9$)
= Pr($Y \leq 3$) by taking the square root of both sides of the inequality
= 0.2+0.2+0.2 = 0.6

**f) Make a probability distribution for $(Y+2)^2$**

Because Y can take values 1,2,3 or 4, Y+2 can take values 3,4,5 or 6. Then $(Y+2)^2$ can take values 9, 16, 25 or 36. The probabilities correspond:

| k | Pr($(Y+2)^2$=k) |
|---|---|
| 9 | 0.2 |
| 16 | 0.2 |
| 25 | 0.2 |
| 36 | 0.4 |