Math 160, Finite Mathematics for Business

Section 7.3: Binomial Trials - Discussion Notes
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Binomial Trials

Binomial trials are a way of modeling a specific family of experiments. In these a certain trial is conducted n times, each time with the same probability p of success. The observed data value is the number of successes. Often We say $q = 1 - p$ is the probability of failure. To compute the probability of exactly k successes, we use the following formula:

$$Pr(X = k) = \binom{n}{k} p^k q^{n-k}$$

We say that X is a **binomial random variable** with n and p as parameters.

To compute $Pr(X \leq k)$ we must add up the cases $Pr(X = 0) + Pr(X = 1) + \cdots + Pr(X = k)$.

ex) A Die is rolled 9 times. What is the probability of rolling only one 6?

Because the probability of rolling a 6 is $\frac{1}{6}$ and we are repeating the roll 9 times, this is a binomial trial. Using the binomial trials formula with $n = 9, p = \frac{1}{6}$, and $k = 1$, we have $Pr(X = 1) = \binom{9}{1} \frac{1}{6}^1 \frac{5}{6}^8 \approx .3489$

ex) A basketball player makes free-throws 80% of the time. What is the probability he makes exactly 3 out of 5 shots?

In this example, n=5, p=.80 and we want to know $Pr(X = 3)$. This is $\binom{5}{3}.80^3 \cdot .20^2 \approx .2048$

ex) 19$ of the population uses a certain band of detergent. From a sample of 15 shoppers, what is the probability that more than 2 use this brand?

To calculate $Pr(X > 2)$ we would have to add up all cases $Pr(X = 3) + Pr(X = 4) + \cdots Pr(X = 15)$ which would be a tedious calculation. Instead we can use the complement rule: The probability more than 2 use the brand is 1 minus the probability that less than 3 use the brand; i.e. $Pr(X > 2) = 1 - Pr(X = 0$ or $X = 1$ or $X = 2) = 1 - [\binom{15}{0}.19^0.81^{15} + \binom{15}{1}.19^1.91^{14} + \binom{15}{2}.19^2.81^{13}] \approx .5635$

ex) For a 5 day vacation in Aspen with a 30% chance of snow each day, create a probability distribution for all possible numbers of snowy days.

k: # Snow Days	$Pr(X = k)$
0	$\binom{5}{0}.3^0.7^5 \approx .168$
1	$\binom{5}{1}.3^1.7^4 \approx .360$
2	$\binom{5}{2}.3^2.7^3 \approx .309$
3	$\binom{5}{3}.3^3.7^2 \approx .132$
4	$\binom{5}{4}.3^4.7^1 \approx .028$
5	$\binom{5}{5}.3^5.7^0 \approx .002$

What is the probability of getting 3 or more snow days?

We can add up the probabilities from the distribution: $.132 + .028 + .002 = .162$