Math 160, Finite Mathematics for Business

Section 7.4: Mean and Expected Value - Discussion Notes
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Population: The set of all elements about which information is desired.
Sample: A subset of the population that is analyzed in an attempt to estimate certain properties of the entire population.
We often do not have the resources, money, or time to determine exact parameters of the population, so we take samples and calculate statistics on the sample. A common and useful statistic is the **mean** or **average**.
Sample Mean: Given a sample of data $x_1, x_2, ..., x_n$, the sample mean $\bar{x}$ is calculated as follows:

$$\bar{x} = \frac{x_1+x_2+\cdots+x_n}{n}$$

If you are given the raw frequencies $f_1, f_2, ..., f_r$ of the outcomes $x_1, x_2, ..., x_r$, you may calculate sample mean by:

$$\bar{x} = \frac{x_1(f_1)+x_2(f_2)+\cdots+x_r(f_r)}{n} = x_1(\frac{f_1}{n}) + x_2(\frac{f_2}{n}) + \cdots + x_r(\frac{f_r}{n})$$

The **population mean** μ is calculated using the same formula.

Expected value of a random variable is the same as population mean. If you are given a probability distribution for a random variable X,:

$\mathbf{x_i}$	$\mathbf{Pr(X = x_i)}$
x_1	p_1
x_2	p_2
$\vdots$	
x_N	p_N

Expected Value of X, written $E(X) = x_1(p_1) + x_2(p_2) + \cdots + x_N(p_N)$

If X is a binomial random variable, with parameters n and p, $E(X) = np$.

7.4.7) Roulette: In American roulette, the wheel has 38 numbers that you can bet on. Let's suppose the bet is $1. If your number comes up, you win $35 plus you keep your bet. If your number does not come up you lose your bet. What is the expected value of a game of American Roulette?
The probability of a win is $\frac{1}{38}$ and a loss is $\frac{37}{38}$, while the value of a win is $35 and the value of a loss is -$1.
So the Expected value is $35(\frac{1}{38}) - 1(\frac{37}{38}) \approx -0.0526$
This means that each game of roulette will average a loss of just over 5 cents.

7.4.9) Carnival Game: A game at the carnival costs $1 to play. There is an urn containing 2 red balls and 4 white balls. You draw balls from the urn until you pull out a red ball at which point the game is over. You receive $0.50 per white ball you have drawn. What is the expected value of the game?
Let's first create a probability distribution for this game.

Outcome	Net $	Probability
R	-1.00	$\frac{2}{6} = \frac{1}{3}$
WR	-.50	$\frac{4}{6}\frac{2}{5} = \frac{4}{15}$
WWR	0.00	$\frac{4}{6}\frac{3}{5}\frac{2}{4} = \frac{1}{5}$
WWWR	+0.50	$\frac{4}{6}\frac{3}{5}\frac{2}{4}\frac{2}{3} = \frac{2}{15}$
WWWWR	+1.00	$\frac{4}{6}\frac{3}{5}\frac{2}{4}\frac{1}{3}\frac{2}{2} = \frac{1}{15}$

$E(X) = -1(\frac{1}{3}) - .5(\frac{4}{15}) + 0(\frac{1}{5}) + .5(\frac{2}{15}) + 1(\frac{1}{15}) = -0.333$

7.4.10) Coins in a bag: In this carnival game, there is a bag containing 2 silver dollars and 6 "slugs" (fake coins that feel real). You reach into the bag and take out two coins. You receive $1 for each silver dollar you pull out. What is a fair price for this game?
A fair price would be the same as the expected gain - thus you would on average break even. So to answer the question, we need to calculate the expected value.

Gain	Probability
$0	$\binom{6}{2}/\binom{8}{2} = \frac{15}{28}$
$1	$\binom{6}{1}\binom{2}{1}/\binom{8}{2} = \frac{3}{7}$
$2	$\binom{2}{2}/\binom{8}{2} = \frac{1}{28}$

Expected value $= 0(\frac{15}{28}) + 1(\frac{3}{7}) + 2(\frac{1}{28}) = .5$. So a fair price would be $0.50 to play this game (but knowing carnies, that's unlikely - you'll probably be charged a dollar).

7.4.12) Life Insurance: A husband and wife are buying a life insurance policy. Based on their health and age, they predict the man has a 90% of living at least 5 more years, and 95% for the woman. The 5-year insurance policy will pay $10,000 if one spouse dies during the period of insurance and $15,000 if both die during the period. What is a fair price for this policy?
We need to calculate expected value based on the probability distribution. The table is as follows:

Outcome	Payout	Probability
Both live	$0	$.90 \times .95 = .855$
Man dies	$10,000	$.10 \times .95 = .095$
Woman dies	$10,000	$.90 \times .05 = .045$
Both die	$15,000	$.10 \times .05 = .005$

Expected value $= 0(.855) + 10000(.095 + .045) + 15000(.005) = 1475$, so a fair price for the policy would be $1,475

7.4.13) Pair of dice: You roll two dice and observe the higher of the two numbers rolled. What is the expected value?
The possible outcomes in this experiment are the numbers 1 through 6, but they do not have equal probability. The only way for 1 to be the highest number rolled is for both dice to have rolled a 1, and the probability is $\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$. For a 2 to be the highest number rolled, this will occur if you roll (1,2), (2,1) or (2,2), so we have the probability of $\frac{3}{36}$. By similar reasoning, you can create the probability distribution:

Outcome	Probability
1	$\frac{1}{36}$
2	$\frac{3}{36}$
3	$\frac{5}{36}$
4	$\frac{7}{36}$
5	$\frac{9}{36}$
6	$\frac{11}{36}$

Expected value is calculated to be 4.47