Math 160, Finite Mathematics for Business
Section 7.5: Variance and Standard Deviation – Discussion Notes
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Variance measures how spread away the data is from the mean – smaller spread means a smaller variance. We use Var(X) to mean the variance of random variable X.
If $E(X)=\mu$, $\mathbf{Var(X)=E((X-\mu)^2)}$

Given a probability distribution:

x_i	P(X=xi)
x_1	p_1
x_2	p_2
⋮	⋮
x_n	p_n

Then $\mathbf{Var(X)=(x_1-\mu)^2p_1+(x_2-\mu)^2p_2+...+(x_n-\mu)^2p_n}$
An alternative formula is $\mathbf{Var(X) = E(X^2)-\mu^2 = [\, x_1^2p_1 + x_2^2p_2 + \dots + x_n^2p_n\,] - \mu^2}$

ex) Compute the variance:

outcome	**probability**
70	**0.5**
71	**0.2**
72	**0.1**
73	**0.2**

First of all, $\mu = E(X) = 70(0.5)+71(0.2)+72(0.1)+73(0.2) = 71$
$Var(X)=(70-71)^2(0.5)+(71-71)^2(0.2)+(72-71)^2(0.1)+(73-71)^2(0.2)=.5+.1+.8=1.4$

ex) Compute variance from alternative formula

outcome	**probability**
2	**0.1**
3	**0.3**
4	**0.5**
5	**0.1**

$\mu = E(X) = 2(0.1)+3(0.3)+4(0.5)+5(0.1) = 3.6$
$Var(X)= (2^2)(.1)+(3^2)(.3)+(4^2)(.5)+(5^2)(.1)-.36^2 = 0.64$

The **standard deviation** is the square root of the variance. We use sigma (σ) to represent the standard deviation. $\sigma_x=\sqrt{Var(X)}$.

For the Binomial Model, X~Binom(n,p) we have Var(X) = npq = np(1-p) and $\sigma_x=\sqrt{npq}$

Chebychev's Inequality

For a probability distribution with $E(X)=\mu$ and Standard Deviation σ,

$$\Pr(\mu-c < X < \mu+c) \geq 1- \sigma^2/c^2$$

This gives us lower bound for the probability that X falls within a range around the mean. This lower bound is true even if we know nothing else about the probability distribution (skewness, for example).

From Chebychev's Inequality, we can say that the probability that a random variable is within:
2 standard deviations of the mean is at least 3/4
3 standard deviations of the mean is at least 8/9
4 standard deviations of the mean is at least 15/16

ex) A probability distribution has mean 75 and std. Dev 12. For what value of c is
$\Pr(75\text{-}c< X <57+c) \geq 9/25$?

$\frac{9}{25}=1-\frac{\sigma^2}{c^2}=1-\frac{144}{c^2}$ then $\frac{144}{c^2}=\frac{25}{25}-\frac{9}{25}=\frac{16}{25}$ and $144\cdot 25=16c^2$ and $c=15$

ex) Light bulbs work for 4000 hours average, with standard deviation of 250 hours. If you install 6000, estimate how many will burn out between 3,250 and 4,750 hours.

First of all, what is the probability that a single bulb lasts in that range?

$\Pr(3{,}250 < X < 4{,}750) = \Pr(4000 - 750 < X < 4000 + 750) \geq 1- \sigma^2/c^2 = 1 - 250^2/750^2 = 8/9$

So for 6000 bulbs, 6000*8/9 is about 5,333. We estimate at least 5,333 bulbs will burn out within this range.

ex) Electronics are packaged in boxes of 300 units. They have a probability of .013 of being defective. What is the mean and standard deviation of defective units per box?

This can be modeled as a Binomial, since each box has 300 units, each unit is like a Bernoulli trial with a probability of 0.013 of being defective.

So X~Binomial(300,0.013), n=300 and p=0.013

So $\mu=E(X)=n*p=300*0.013 = 3.9$

and $\sigma= \sqrt{npq} = \sqrt{300*.013*.987} = 1.962$