**Math 160 Discussion Notes**
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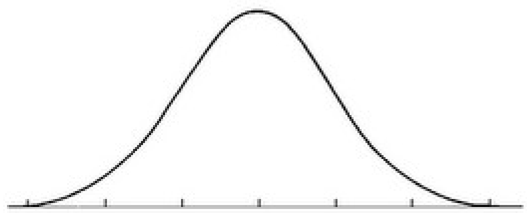
**7.6 The Normal Distribution**

The Normal distribution is a continuous probability distribution, as opposed to a discrete distribution. A Continuous random variable can take any real value along an interval, while a discrete random variable can only take specific values.

ex) The exact height of a random UIC student is a continuous random variable, while the shoe size is discrete.

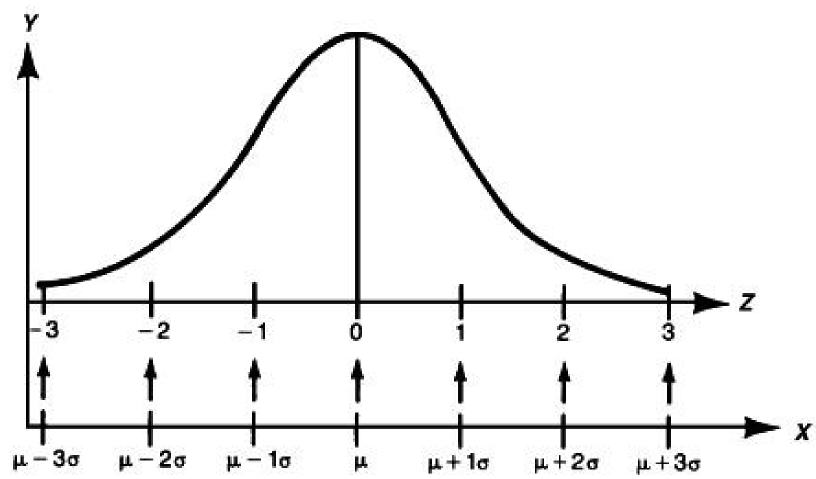
The Normal distribution is a very common distribution and incredibly useful in statistics. Although we cannot draw a histogram for a continuous probability distribution, we can illustrate its probability density function which is analogous.



This is a typical normal curve. The normal curve is entirely characterized by two parameters: mean and standard deviation. The mean gives the center of the curve (where its peak is) and the standard deviation describes how flat/wide or narrow/tall the curve is. We often will use the notation $X \sim N(\mu,\sigma)$ to mean "X is a normal random variable with mean $\mu$ and standard deviation $\sigma$".

A Standard Normal has mean $\mu=0$ and $\sigma=1$. We use Z to represent a standard normal random variable. We can always standardize or un-standardize using the following formulas:

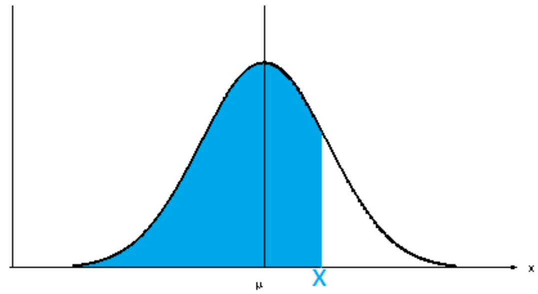
$$z=\frac{x-\mu}{\sigma} \quad \text{or} \quad x=(z\cdot\sigma)+\mu$$




The Translation of $X$ to $Z$ by the Transformation $Z = (X-\mu)/\sigma$

*Figure 3*

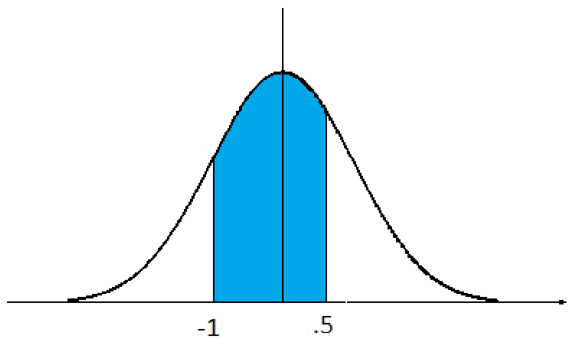
For the area under the curve between two values is the probability that the random variable is within that interval. So if I want to know $P(X<5)$




Where X~N(4,2) this is the same as $P(Z<\frac{5-4}{2})=P(Z<.5)=.6915$

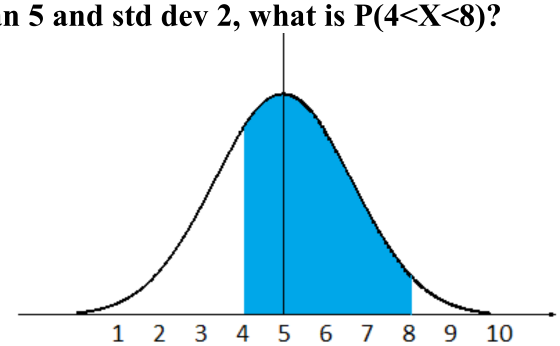
You can find the probability of a standard normal from a Z-Table.

**ex) What is P(-1<Z<.5)?**




The Z-table gives us the area to the left of .5 is .6915, and the area to the left of -1 is .1587. Thus the area between them is .6915-.1587=.5328

**ex) If X is normal with mean 5 and std dev 2, what is P(4<X<8)?**




We can standardize 4 and 8:

$$Pr(4<X<8)=Pr(\frac{4-5}{2}<Z<\frac{8-5}{2})=Pr(-.5<Z<1.5)=Pr(Z<1.5)-Pr(Z<-.5)\approx .6247$$

**Percentiles**

If a value S is the **pth percentile**, that means p% of the data falls below S, (100-p)% is above S.
In other words, the probability of being less than S is p%.

**ex) X is normal with mean 5 and std dev 2, what is the 83$^{rd}$ percentile?**

We want to find x such that $Pr(X<x)=.83$ . Thus $Pr(Z<\frac{x-5}{2})=.83$ . From the z-table, we find that z=.9542 is such that $Pr(Z<.9542)=.83$ , so $\frac{x-5}{2}=.9542$ , so x=6.9084.