Math 160 Practice Final Solutions - Powers Fall 2011

$\boxed{1}$

D

$\mu = 62.8\text{cm}$

$\sigma = 8.4\text{cm}$

$Pr(X \geq 73.3\text{cm})$

$= \text{normalcdf}(73.3, 1000000, 62.8, 8.4) = .1056$

$\boxed{2}$

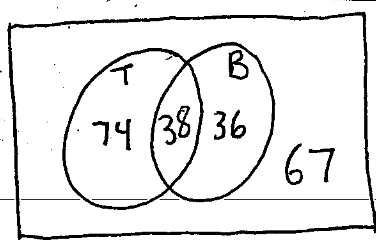
C

74 - Bus

112 - Trains

38 - Both

67 - Neither



Sum is 215

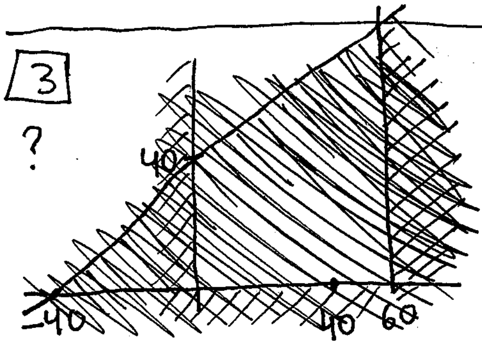
$n(B \cup T) = n(B) + n(T) - n(B \cap T) = 148$

$+ 67$

215

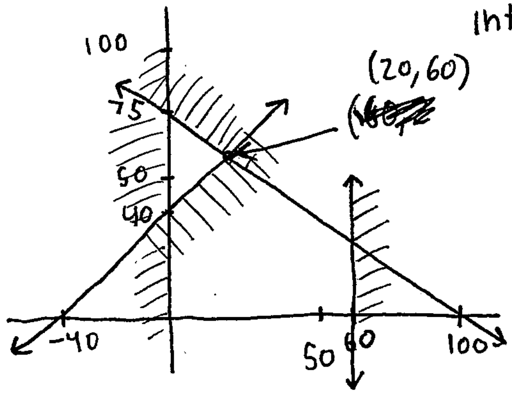
$\boxed{3}$

?



$-x + y \geq 40$ x intercept −40, y intercept 40, (0,0) not included

$3x + 4y \leq 300$ x intercept: 100, y intercept: 75, (0,0) included



Intersection of

$$\begin{cases} 3x + 4y = 300 \\ -x + y = 40 \end{cases}$$

add $R_1 + 3R_2$

$\hookrightarrow 7y = 420$

$y = 60$

$x = \cancel{}\ 20$

(x, y)	$-2x + 5y$
$(0, 40)$	200
$(0, 75)$	375
$(20, 60)$	260

Max is 375 at $(0, 75)$

$\boxed{4}$

C

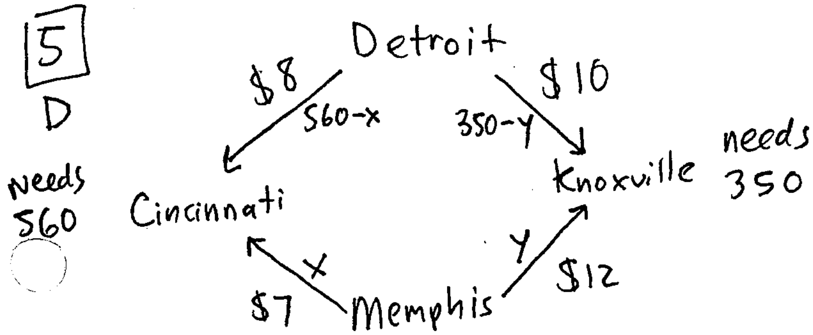
$A = \begin{bmatrix} 4 & -1 \\ 2 & .5 \end{bmatrix}$ $A^{-1} = \frac{1}{4 \cdot .5 + 2} \cdot \begin{bmatrix} .5 & 1 \\ -2 & 4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} .5 & 1 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} .125 & .25 \\ -.5 & 1 \end{bmatrix}$ B not its inverse

$C = \begin{bmatrix} 2P & 0 \\ 4P & -1 \end{bmatrix}$ $C^{-1} = \frac{1}{-2P - 0}\begin{bmatrix} -1 & 0 \\ -4P & 2P \end{bmatrix} = -\frac{1}{2P}\begin{bmatrix} -1 & 0 \\ -4P & 2P \end{bmatrix} = \begin{bmatrix} 1/2P & 0 \\ 2 & -1 \end{bmatrix}$ D is its inverse

$E = \begin{bmatrix} k & -2 \\ -1 & \frac{1}{k} \end{bmatrix}$ $E^{-1} = \frac{1}{k \cdot \frac{1}{k} - 2}\begin{bmatrix} \frac{1}{k} & 2 \\ 1 & k \end{bmatrix}$ $1 = -\begin{bmatrix} \frac{1}{k} & 2 \\ 1 & k \end{bmatrix} = \begin{bmatrix} -\frac{1}{k} & -2 \\ -1 & -k \end{bmatrix}$ F is not its inverse

$\boxed{5}$

D



$$\begin{aligned}\text{Minimize cost} &= 7x + 12y + 8(560-x) + 10(350-y)\\ &= 7x + 12y + 4480 - 8x + 3500 - 10y\\ &= 7980 - x + 2y\end{aligned}$$

$\boxed{6}$

a

Committe of 7 from 8 Rep & 12 Dem

$$\Pr(4,5,6 \text{ or } 7 \text{ Republicans}) = \frac{\binom{8}{4}\binom{12}{3} + \binom{8}{5}\binom{12}{2} + \binom{8}{6}\binom{12}{1} + \binom{8}{7}\binom{12}{0}}{\binom{20}{7}}$$

$$\approx .25077$$

$\boxed{}$

c

$$A = \begin{array}{cc} \text{ATT} & \text{NOT} \\ \end{array}$$
$$A = \begin{bmatrix} .7 & .25 \\ .3 & .75 \end{bmatrix} \qquad A^2 = \begin{bmatrix} .565 & .3625 \\ .435 & .6375 \end{bmatrix}$$

so .565 of today's college women's grand daughters will attend college.

$\boxed{8}$

b

$n = 450 \quad p = \frac{2}{3} \quad q = \frac{1}{3}$

$\mu = 450 \cdot \frac{2}{3} = 300$

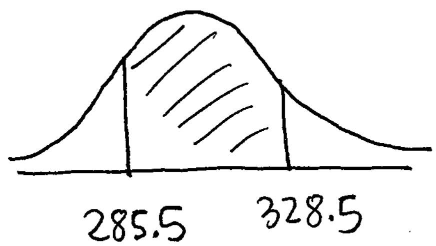
$\sigma = \sqrt{450 \cdot \frac{2}{3} \cdot \frac{1}{3}} = 10$

More than 285

less than or equal to 328

normalcdf(285.5, 328.5, 300, 10)

$= .92428$



9

$$\begin{array}{ccc} & & \text{To Produce} \\ & & \begin{array}{cc} F & C \end{array} \\ \text{needs} & \begin{array}{c} F \\ C \end{array} & \begin{bmatrix} .08 & .04 \\ .12 & .15 \end{bmatrix} = A \end{array}$$

Need $D = \begin{bmatrix} 34,567 \\ 23,456 \end{bmatrix}$

b

$$X = (I-A)^{-1} D = \begin{bmatrix} 39,012.08 \\ 33,102.88 \end{bmatrix}$$

Total is $72,114.96 \approx 72,115.00$

10

a

$P(A) = .4 \Rightarrow P(A') = .6$

$P(B') = .48$

$P(A \cup B) = .64$

so $P(A' \cap B') = .36$ by deMorgan's law

$(= P((A \cup B)'))$

$$P(A' | B') = \cancel{\frac{P(A')}{P(A)}} \; \frac{P(A' \cap B')}{P(B')} = \frac{.36}{.48} = .75$$

11

a

$$\begin{bmatrix} 2 & 3 & 0 & -29 \\ 1 & 1 & -1 & -12 \\ 2 & -3 & -12 & 1 \end{bmatrix} \xrightarrow{\text{rref}} \begin{bmatrix} 1 & 0 & -3 & -7 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x - 3z = -7 \Rightarrow x = 3z - 7$

$y + 2z = -5 \quad y = -2z - 5$

$z =$ any real

---

12 d

	X Tables	Y Chair	Available
Carpentry	4	2	360
Sanding	2	1	120
Finishing	3	1	180

$4x + 2y \leq 360$

$2x + y \leq 120$

$3x + y \leq 180$

AT MOST 3 times as many chairs as tables

$x \leq 3y$

so $x - 3y \leq 0$

13

d

11 possible Toppings

Up to 8 toppings = not 9, 10 or 11

$$2^{11} - \binom{11}{9} - \binom{11}{10} - \binom{11}{11} = 1981$$

$\boxed{14}$

$$\frac{7\cdot 4 + 6\cdot 3 + 8\cdot 2 + 5\cdot 1 + 2\cdot 0}{28} = 2.3929$$

$\boxed{15}$

b

$BDC + E \qquad [2\times 3]\underbrace{[3\times 3][3\times 2]}_{2\times 2} + [2\times 2] \qquad + 2\times 2 \quad$ ok

$CAB + CD \qquad [3\times 2][2\times 3][2\times 3]$

$\uparrow$ NOT OK

$A+B+C \qquad [2\times 3] + [2\times 3] + [3\times 2]$

$\uparrow$ NOT OK

$EA + BCB \qquad \underbrace{[2\times 2][2\times 3]}_{2\times 3} + \underbrace{[2\times 3][3\times 2][2\times 3]}_{2\times 3} \quad$ ok

I and IV

$\boxed{16}$

c

$$S(I-R)^{-1} = \begin{bmatrix} .5 & .3 \\ .2 & .2 \end{bmatrix}\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .2 & .4 \\ .1 & .1 \end{bmatrix}\right)^{-1} = \begin{array}{c} \\ \\ \end{array}\begin{array}{cc} P & S \end{array}$$

or take ~~A~~ A^{100} and look at $\begin{array}{c} \begin{array}{cccc} Q & R & P & S \end{array} \\ \begin{bmatrix} & & \square & \\ & & & \end{bmatrix} \end{array}\begin{array}{c} Q \\ R \\ P \\ S \end{array}$

$$= \begin{bmatrix} .70588 & .64705 \\ .29412 & .35294 \end{bmatrix}\begin{array}{c} Q \\ R \end{array}$$

$\boxed{17}$

a

$P(\text{Pos} \mid \text{Infected}) = .94 \qquad P(\text{Infected}) = .20$

$P(\text{Neg} \mid \text{Infected}) = .06$

$P(\text{Neg}) = \cancel{.97 \times .20 + .06} \times$

$P(\text{Pos} \mid \text{Not Infected}) = .03$

$.06 \times .20 + .97 \times .80$

$P(\text{Neg} \mid \text{Not Infected}) = .97$

$= .788$

$$P(\text{Infected} \mid \text{Neg}) = \frac{P(\text{Infected})\,P(\text{Neg} \mid \text{Infected})}{P(\text{Inf.})\,P(\text{Neg} \mid \text{Inf}) + P(\text{Not Inf})\,P(\text{Neg} \mid \text{Not Inf})} = \frac{.2 \times .06}{.2(.06) + .8(.97)} = .0152$$

18

d

16 into $(5,4,3,2,1,1)$

$$= \frac{16!}{5!\,4!\,3!\,2!} = 605{,}404{,}800$$

19

c

Investment A

$$\mu = -3000(.2) + 1500(.2) + 4000(.45)$$
$$= 1500$$
$$\sigma^2 = (-4500)^2 \cdot .2 + (-1500)^2 \cdot .15 + (2500)^2 \cdot .45$$
$$= 7{,}200{,}000$$

Investment B

$$\mu = 1000(.15) + 1500(.5) + 2000(.3)$$
$$= 1500$$
$$\sigma^2 = (-1500)^2 \cdot .05 + (-500)^2 \cdot .15 + (500)^2 \cdot .3$$
$$= 225{,}000$$

means equal

Standard dev of A > std dev of B

20

c

$$\cancel{\begin{array}{cc|c} \text{Heavy} & \text{Light} & \\ .3 & & H \\ & & L \end{array}}$$

$$A = \begin{array}{c} \begin{array}{ccc} H & L & \text{None} \end{array} \\ \begin{bmatrix} .3 & .35 & 1 \\ .5 & .4 & 0 \\ .2 & .25 & 0 \end{bmatrix} \end{array} \begin{array}{l} H \\ L \\ \text{None} \end{array}$$

Not Absorbing

$$A^{100} = \begin{bmatrix} .446 & \cdots & \cdots \\ .372 & \cdots & \cdots \\ .182 & \cdots & \cdots \end{bmatrix}$$