Math 160 , Finite Mathematics for Business

Section 5.3-5.8 Review Problems - Discussion Notes
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1) If you have a deck of 52 cards and are dealt a hand of 5 cards, in how many ways can you be dealt 2 or fewer red cards?

Whenever the question uses a phrase like "2 or fewer" it should be a signal that you have to count individual cases. In this instance, we have to consider cases of "exactly 2 red cards", "exactly 1 red card" and "exactly 0 red cards". To count the ways the first case can occur, we need to account for all 5 cards in the hand. There are 26 red cards in the deck and 26 black cards. So there are $\binom{26}{2}$ possibilities for the two red cards and $\binom{26}{3}$ possibilities for the 3 black cards. We take the product to get the number of hands:
Ways to get 2 red cards: $\binom{26}{2} \cdot \binom{26}{3} = 845,000$
Ways to get 1 red card: $\binom{26}{1} \cdot \binom{26}{4} = 388,700$
Ways to get 0 red cards: $\binom{26}{0} \cdot \binom{26}{5} = 65,780$
The sum of these three counts is 1,299,480, which is our answer.

2) If you're planning a 7 day dream vacation in Costa Rica and hope for at least 5 sunny days, in how many ways can you experience your dream vacation?

Once again, we need to consider the disjoint cases which satisfy the criteria. At least 5 sunny days means "5 sunny days or 6 sunny days or 7 sunny days". To describe the weather of your vacation, you would say something like "It rained on the second, third and fifth days" or "It rained on the first day and the last day". If you wanted to count all possible vacations in which there were 5 sunny days, you would count how many possible combinations of 5 days of the week you could choose to be sunny. This is $\binom{7}{5} = 21$. Likewise, to count the number of vacations with 6 sunny days, we count $\binom{7}{6} = 7$. The number of vacations with all 7 sunny days is $\binom{7}{7} = 1$. The sum of these three numbers is 29.

3) How many distinct 5 letter "words" can you make by rearranging the letters in the word "MOOSE" ?

If the 5 letters were different from each other, then there would be 5! permutations of the 5 letters. However, we have two Os, and we have to be careful to not count the same word twice.
The way to solve this problem is to think of the rearrangements of letters as an ordered partition. The 5 letters in our new word can be indexed by the numbers 1-5. We assign these five numbers to the letters of MOOSE by partitioning. One goes to M, two go to O, one to S and one to E. This is an ordered partition of type (1,2,1,1), and the formula for this is $\frac{5!}{1!2!1!1!} = \frac{120}{2} = 60$. There are 60 distinct words that can be formed from these 5 letters.

3b) How many distinct 9 letter "words" can you make by rearranging the letters in the word "REARRANGE" ?

Using the same method as in the previous problem, we can think of each word as an ordered partition of the letters. The partitions are: $3Rs, 2Es, 2As, 1Nand1G$. So we need to count ordered partitions of type (3, 2, 2, 1, 1). This is $\frac{9!}{3!2!2!} = 15,120$.

4) There are 25 students in class. 5 are wearing both a UIC sweatshirt and sunglasses (even though class is indoors). 7 students are not wearing sunglasses. Half as many students are wearing UIC sweatshirts as are wearing sunglasses. How many students are not wearing a UIC sweatshirt?

Let's start by defining two sets: S={students wearing sweatshirts} and G={students wearing sunglasses}.
5 students are in both sets, so $n(S \cap G) = 5$.
7 students are not wearing sunglasses, which means 25-7=18 are wearing sunglasses. $n(G) = 18$.
Half as many are wearing sweatshirts as sunglasses. This means $n(S) = \frac{1}{2}n(G)$, so $n(S) = 9$.

Because there are 25 students and 9 are wearing sweatshirts, 25-9=16 are not wearing sweatshirts.