Math 180: Calculus I	**Fall 2014**

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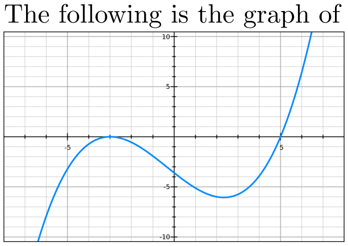
At this point you should have the following derivatives memorized.

$f(x)$	$\frac{d}{dx}f(x)$	comments
$g(x)+h(x)$	$g'(x)+h'(x)$	Sum Rule
$g(x)-h(x)$	$g'(x)-h'(x)$	Difference Rule
$g(x)h(x)$	$g'(x)h(x)+g(x)h'(x)$	Product Rule
$g(x)/h(x)$	$(g'(x)h(x)-g(x)h'(x))/h(x)^2$	Quotient Rule
$g(h(x))$	$g'(h(x))h'(x)$	Chain Rule
c	0	for any constant c
x^n	nx^{n-1}	Power Rule
e^x	e^x	
b^x	$b^x \ln b$	
$\ln x$	$\frac{1}{x}$	
$\log_b x$	$\frac{1}{x \ln b}$	
$\sin x$	$\cos x$	Trig Functions
$\cos x$	$-\sin x$	
$\tan x$	$\sec^2 x$	
$\sec x$	$\sec x \tan x$	
$\cot x$	$-\csc^2 x$	
$\csc x$	$\csc x \cot x$	
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	Inverse Trig Functions
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	
$\tan^{-1} x$	$\frac{1}{x^2+1}$	
$\cot^{-1} x$	$\frac{-1}{x^2+1}$	

Important techniques and terms:
Implicit Differentiation: take the derivative of both sides of the equation, treat y as a function of x, and then solve for y'
Logarithmic Differentiation: First take a ln or log of both sides of the equation, then solve for y' using implicit differentiation.
Critical Points: values of x in the domain where $f'(x) = 0$ or $f'(x)$ is undefined.
Local Extrema: cannot be points of discontinuity - locally a maximum or a minimum.
Global Extrema: The absolute maximum and minimum values the function takes on the domain or an interval - could be at an endpoint.
Concavity: The curvature of a graph - concave up means $f''(x) > 0$, concave down when $f''(x) < 0$.
Inflection Point: An x value that separates an interval of upward concavity from an interval of downward concavity. Inflection point implies $f''(x) = 0$.
2nd Derivative Test: If a is a critical point, $f''(a) > 0 \Rightarrow a$ is a local min, $f''(x) < 0 \Rightarrow a$ is a local min.

Practice Problems

1. Find the derivatives of the following functions
   (a) $f(x) = (\log_{10} x)/10^x$
   (b) $f(x) = \cos^{-1}(5x)$
   (c) $f(x) = \cos(x^{\cos x})$
   (d) $f(x) = 3x \cdot \tan^{-1}(x^3)$
   (e) $f(x) = (\sin x)^x$
2. Two real numbers x and y have a product of 15. What is the minimum of $3x+5y$? Is there a maximum value of $3x + 5y$?
3. The following is the graph of $f'(x)$.



   (a) Determine the critical points of $f(x)$.
   (b) Determine the intervals on which $f(x)$ is increasing or decreasing.
   (c) Classify each critical point as a local min, local max or neither.
   (d) Determine (approximately) the intervals on which $f(x)$ is concave up, concave down and any points of inflection.
4. Consider $g(x) = 9x^{1/3} - 4$.
   (a) Find the second derivative, $g''(x)$.
   (b) Find the intervals on which $g(x)$ is concave up.
   (c) Locate any inflection points of $g(x)$.
5. Consider the function $g(x) = x\sqrt{x+1}$.
   (a) State the domain of g.
   (b) State the intervals on which g is increasing and those on which it is decreasing.
   (c) Find all local extrema of g, or state that none exist.
   (d) Sketch $g(x)$ using your analysis.
6. Let $f(x) = xe^{4x}$ on the interval $[-3, 0]$. Find the absolute maximum and minimum values of the function on this interval.
7. A rectangular farm needs to fence in a 20,000 ft^2 rectangle of grass. On one side there is a river. If fencing has a fixed cost per foot, what dimensions will minimize cost?
8. A box has a square base with side lengths x and a constant surface area of $12m^2$.
   (a) Express its volume V as a function of x.

(b) Find the maximum volume of such a box.

9. Consider the curve given by the equation $x \ln x + y \ln y = 1$. Find the equation of the line tangent to point $(e, 1)$.

10. Find the point on the curve $y = x^2/6 + 4$ that is nearest to the point $(0, 13)$.

11. The continuous function $f(x)$ has critical points at $x = 2, 5, 6$, has a local minimum at $x = 2$, a local maximum at $x = 5$ and $f'(6)$ is undefined. The graph is concave up on the intervals $(-\infty, 5), (5, 6), (6, \infty)$. Sketch a possible graph of $f(x)$.