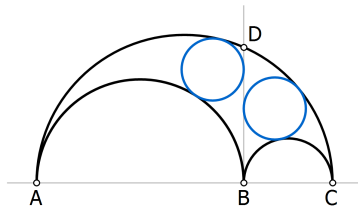


October 30

TA: Brian Powers

- Two nonnegative numbers x and y have a sum of 23. What is the maximum possible product?
- A box's total dimensions (length + width + height) cannot exceed 108 in. If the box has a square base, what is the largest possible volume?
- A rain gutter is formed by taking a sheet of metal 9 in wide and bending it in thirds (making an isosceles trapezoid with an open top). What is the angle that maximizes cross-sectional area?
- A rectangle with its base on the x -axis has two of its vertices on the parabola $y = 16 - x^2$. What are the dimensions that maximize area, and what is the maximum area?
- A piece of wire 60 in. long is cut. One part is bent to make a square, the other is used to make a circle. Where should it be cut to maximize the combined area? Minimize combined area?
- Two right circular cones of height h and radius r are placed on either end of a cylinder of height h and radius r , making a doubly-pointed object. If the surface area is fixed at A , what dimensions h and r will maximize the total volume? (*Hint*: the volume of a cone is $v = \frac{1}{3}\pi r^2 h$, lateral surface area $s = \pi r l$ where l is the length from the base to the vertex. For a cylinder $v = \pi r^2 h$ and lateral surface area $s = 2\pi r h$)
- An arbelos is formed by taking a semicircle and removing two tangent semicircles as so:



Say $AC = 1$. At what position of B will the area of the arbelos be maximized? Show that the area of the arbelos is the same as a circle with diameter BD .