

November 25

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1. Use symmetry to evaluate these integrals

(a) $\int_{-\pi/4}^{\pi/4} \cos x dx$

(b) $\int_{-10}^{10} \frac{x}{\sqrt{200-x^2}} dx$

(c) $\int_0^{2\pi} \sin x dx$

2. Find the average value of the following functions on the interval given

(a) $f(x) = 1/x; [1, e]$

(b) $f(x) = x(1-x); [0, 1]$

3. Find the appropriate point in the interval where the function equals its average value.

(a) $f(x) = e^x; [0, 2]$

(b) $f(x) = 1 - |x|; [-1, 1]$

4. Show that the area of a segment of a parabola is $4/3$ that of the inscribed triangle of greatest area. Specifically, show that the area bounded by $y = a^2 - x^2$ and the x -axis is $4/3$ the area of the triangle with vertices at $(\pm a, 0)$ and $(0, a^2)$. Let $a > 0$ be an arbitrary constant.

5. Use a change of variables (substitution) to find the following integrals

(a) $\int 2x(x^2 - 1)^{99} dx$

(b) $\int x^3(x^4 + 16)^6 dx$

(c) $\int 2x \sin(x^2) dx$

(d) $\int \frac{x^2}{(x+1)^4} dx$

(e) $\int (x+1)\sqrt{3x+2} dx$

(f) $\int_0^1 2x(4-x^2) dx$

(g) $\int_0^{\pi/2} \sin^2 \theta \cos \theta d\theta$

(h) $\int_0^4 \frac{p}{\sqrt{9+p^2}} dp$