# Discrete Probability Distributions

- Bernoulli with parameter p: Bern(p)
- Describes an event with probability "p" of occurring (we call this a "success"). We call $q=1-p$ the probability of "failure"
- $X \sim \text{Bern}(p)$
- $E(X) = p$
- $\text{Var}(X) = p(1-p) = pq$, $SD(X)=\sqrt{(pq)}$

# Bernoulli Example

- You drop your toast and as we all know toast has a 75% chance of landing butter side-down. If it lands butter-side down you need to buy new toast for $1, but if it's butter side up, the 5 second rule applies and you don't have to buy new toast! What is the expected cost and standard deviation of dropping toast?
- $E(X)=p*\$1=\$.75$
- $SD(X)=\sqrt{Var(X)}=\sqrt{(pq)}=\sqrt{.1875}\approx\$.433$

# Multiple Bernoulli Trials

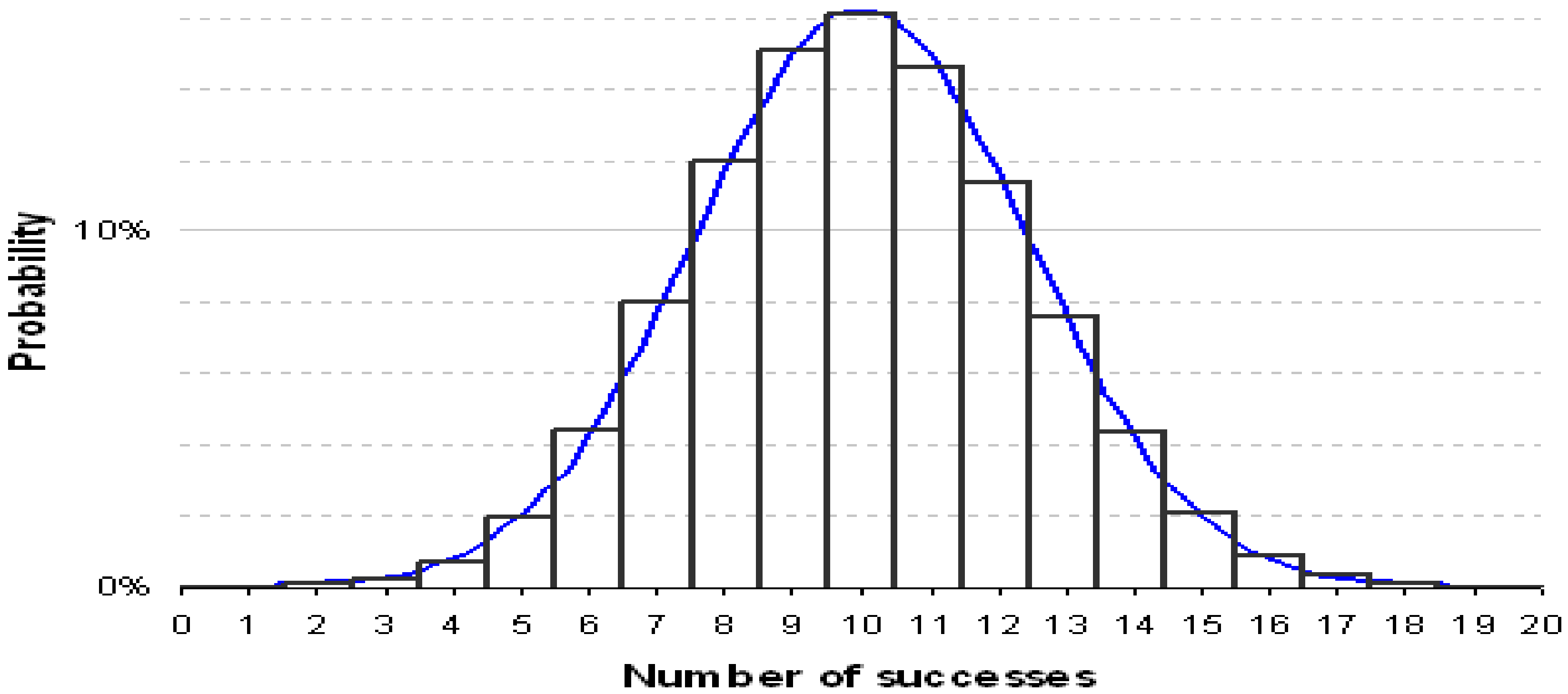
- When we are performing multiple *independent* Bernoulli trials, we make new distributions
- Binomial Distribution: Models the number of successes obtained from "n" independent Bernoulli trials
- Geometric Distribution: Models the number of independent Bernoulli trials until (and including) the first success

# Binomial Distribution

- X~Binom(n,p) means X follows a Binomial model of "n" independent trials with probability "p" of success
- P(X=k) is the probability of "k" successes
- $P(X=k)=C(n,k)*p^{k}*q^{(n-k)}$
- C(n,k) counts all combinations of k out of n (ie., which of the n trials are the k successes)

# Binomial Histogram

- Histogram of Binomial(20, 0.50)



# “n choose k”

- C(n,k) is sometimes written $\binom{n}{k}$
- We say “n choose k”
- The formula is $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
- The TI-83/84 has this built in.
- Go to [MATH]>[PRB] and choose “nCr”
- eg., “8 nCr 3” = 56. This means there are 56 possible combinations of 3 out of 8 things.

# Back to the Binomial

- X~Binom(n,p)
- $P(X=k)=\binom{n}{k}p^k q^{n-k}$
- E(X)=np
- Var(X)=npq, SD(X)=√(npq)
- $P(X\leq k) = P(X=0) + P(X=1) + \cdots + P(X=k)$

# Binomial Example

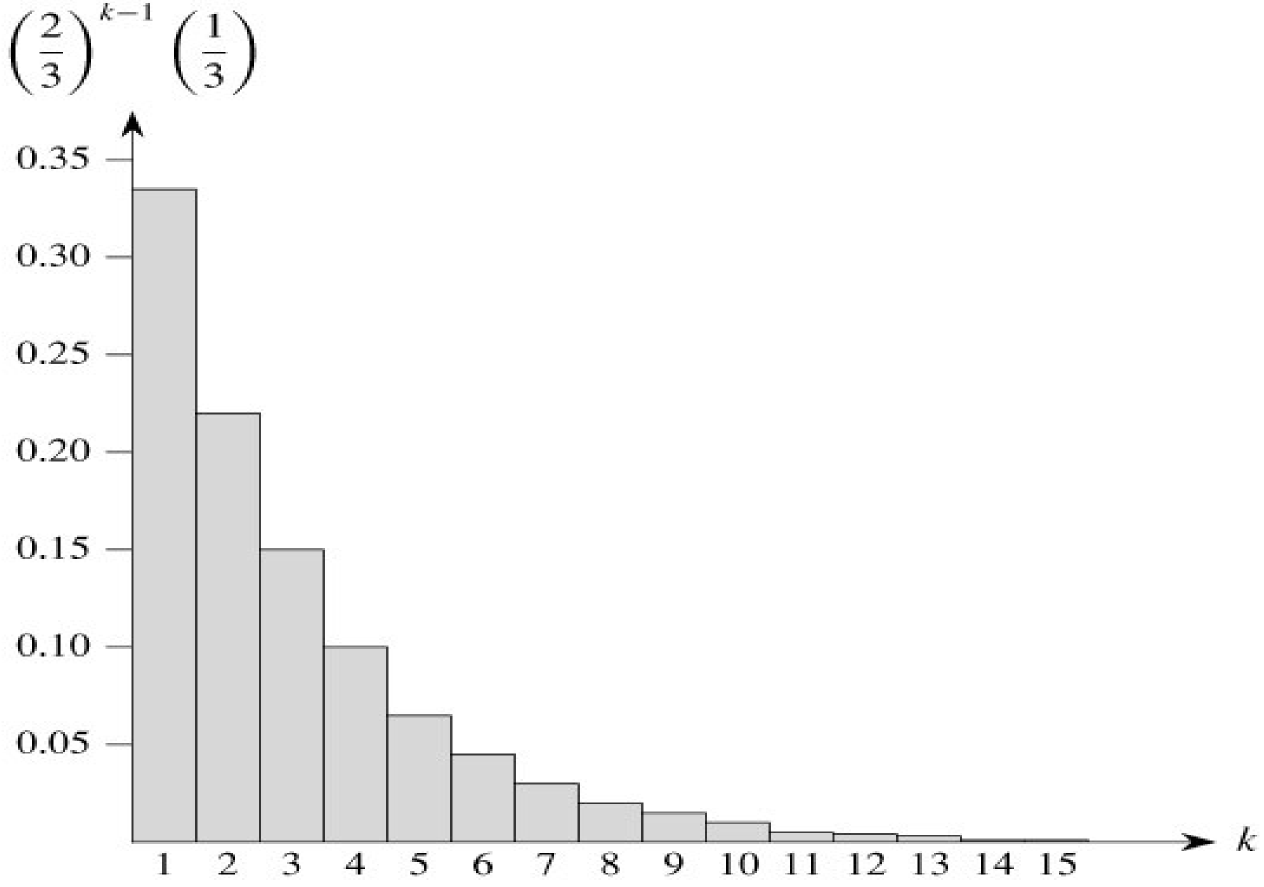
- An archer hits his mark 85% of the time. He fires 5 arrows. What is the probability that 3 of them hit?
- X~Binom(5,.85) because n=5, p=.85
- $P(X=3)=\binom{5}{3}(.85)^3(.15)^2=.13817$
- E(X)=np=5*.85=4.25
- Var(X)=npq=5(.85)(.15)=.6375
- SD(X)=√Var(X)=√.6375=.7984

# Geometric Distribution

- X~Geometric(p) means X follows a Geometric Model with probability “p” of success
- $P(X=k) = p*q^{k-1}$
- ie, the probability of 1 success and k-1 failures
- $E(X)=1/p$
- $Var(X)=q/p^2$, $SD(X)=\sqrt{(q/p^2)}$

# Geometric Histogram

- Histogram of Geom(1/3)



# Geometric Example

- At the apple factory, a barrel of apples has a 4% chance of being spoiled. Your job is to do quality control. What is the expected number of barrels to check until you find a spoiled barrel? What is the Standard Deviation?
- X~Geometric(.04) so E(X)=1/.04=25
- $Var(X)=q/p^2=.96/.04^2=600$ so $SD(X)=\sqrt{600}\approx 24.5$

# Texas Instruments to the Rescue!

- TI-83/84 make use of the Geometric and Binomial models much easier
- [2nd][VARS] gives access to:
  - **`0:binompdf(`**
  - **`A:binomcdf(`**
  - **`D:geometpdf(`**
  - **`E:geometcdf(`**

# How to use TI-83/84 distributions

- **`binompdf(n,p,k)`** gives the probability of exactly k successes out of n trials with probability p
- **`binomcdf(n,p,k)`** gives the probability of k or fewer successes out of n trials
- **`geometpdf(p,k)`** gives the probability that it takes exactly k trials to get a success
- **`geometcdf(p,k)`** gives the probability that it takes k or fewer trials to get a success