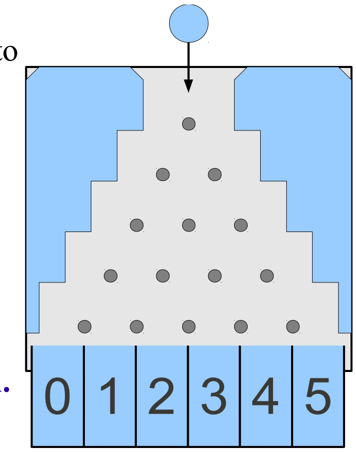
STAT101 – Midterm 2 Review
Fall 2012 - Brian Powers TA
Questions in red are more challenging!



1. You have a board set up like this:
Where a ball is dropped and it hits a peg it is equally likely to go left or right. 6 buckets are set up below numbered 0 - 5.

If we drop 300 balls into this board, how many do we expect to fall into bucket 4?

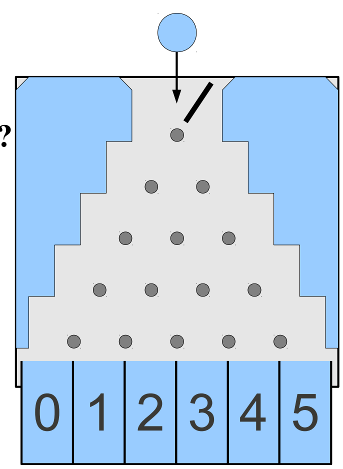
Solution: Each peg the ball hits is like a Bernoulli trial with probability .5 of going left or right. Let's call going right a success. So then the bucket counts the number of 'successes' and should follow Binomial(5, .5) distribution.
P(X=4) is C(5,4)*(.5)^4*(.5)^1 = .15625
But if we drop 300 balls we expect .15625 of them to fall into bucket 4. That is 46.875

What is the mean value for the bucket a ball will land in? What is the standard deviation?

Solution: For Binomial(n,p) the E(X)=np = 5*(.5) = 2.5
Standard Deviation is √(npq)=1.118

**** We keep all balls in their buckets except buckets 0 and 5, which we empty and drop through a second time. Now how what is the <u>total</u> we expect to have in bucket 4?**

Solution: After the first drop, we expect to have 46.875 in bucket 4, and we can calculate that we expect 9.375 in bucket 0 and also 9.375 in bucket 5. Therefore, we expect to drop 18.75 through again, and .15625 of these will expect to fall into bucket 4: that's 2.93 more. So in total, we expect to have 49.805 balls in bucket 4.



You place a stopper at the opening of the board so that the ball is forced to drop left on the first peg.

Now if we drop 300 balls, how many do we expect to fall into bucket 4?

Solution: Because it's impossible for the ball to go right on the first peg, it's impossible for the ball to drop into bucket 5. Effectively, this now models a Binomial(4, .5) distribution.
Therefore, P(X=4) = .0625, and .0625 * 300 = 18.75

What would be the mean value for the bucket, and what would be the standard deviation?

Solution: E(X) = 4(.5) = 2
SD(X) = √(4(.5)(.5)) = 1

2. Given the following:

$P(A) = .5$, $P(B) = .3$, $P(C) = .2$
$P(A|B) = .5$
$P(B \cup C) = .44$
$P(A \cap B \cap C) = .01$
$P(A \cap C) = .05$ (This was missing from the original problem set – it's needed to answer the last question)

What is $P(A \cap B)$?
Solution: $P(A \cap B) = P(A|B) * P(B) = .5 * .3 = .15$

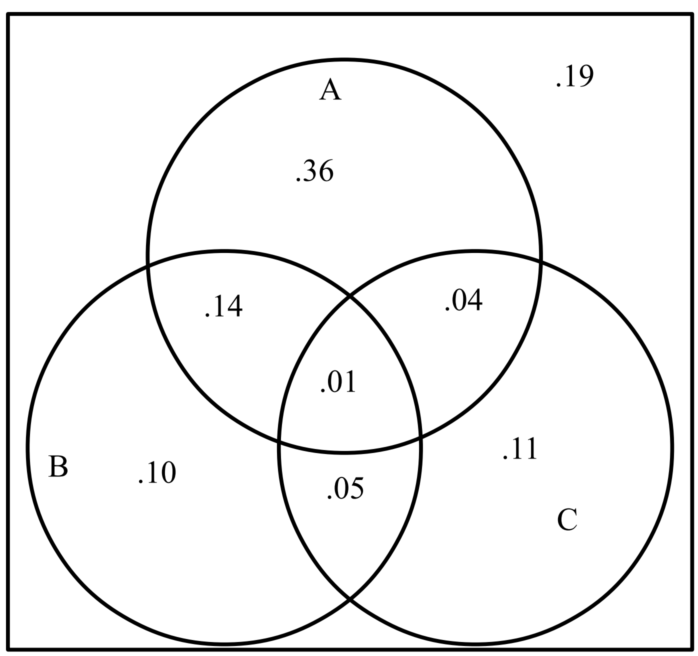
Are B and C independent?
Solution: If B and C are independent then $P(B \cap C) = P(B) * P(C)$
Check: $P(B) * P(C) = .3 * .2 = .06$
We know $P(B \cup C) = .44$, and $P(B \cup C) = P(B) + P(C) - P(B \cap C)$. By doing a little algebra we compute that $P(B \cap C) = .06$.
Yes, B and C are independent.

Are A and B disjoint?
Solution: If A and B were disjoint, that would mean $P(A \cap B) = 0$. But If that were the case, $P(A|B)$ would be 0 also, and that's not true, so it's impossible for A and B to be disjoint.

What is $P(B \cap C^C)$?
Solution: We can always write $P(B) = P(B \cap C) + P(B \cap C^C)$
So $0.30 = .06 + P(B \cap C^C)$. Therefore, $P(B \cap C^C) = .24$

**** What is $P((A \cup B \cup C)^C)$?**

We can begin by filling in .01 for $P(A \cap B \cap C)$
Because we know $P(A \cap B)$ is .15 we can fill in the rest of its intersection with .14
We have already computed that $P(B \cap C) = .06$ so we can fill in .05 in the rest of its intersection
The rest of B must have probability .10
Because $P(A \cap C) = .05$, the rest of its intersection is .04
We can fill in the rest of A and C accordingly with .36 and .11 respectively
The remaining probability must be .19 because it all must add up to 1



3. You have a group of students – some have sweatshirts on, some have glasses. Let S be the event "the student is wearing a sweatshirt" and G be the event "the student has glasses". How do you express the following in Probability notation:

- **The probability a student is wearing a sweatshirt**

$P(S)$

- **The probability a student has glasses or is not wearing a sweatshirt**

$P(G \cup S^C)$

- **The probability that a student wearing a sweatshirt is wearing glasses**

$P(G|S)$

- **The probability a student is wearing glasses and a sweatshirt**

$P(S \cap G)$

- **The probability a student is wearing glasses but no sweatshirt OR a sweatshirt but no glasses**

$P((G \cap S^C) \cup (S \cap G^C))$

How can we check if the S and G are independent?

Answer: If $P(S \cap G)=P(S)*P(G)$

How can we check if S and G are disjoint?

If $P(S \cap G)=0$

4. You draw cards from a deck until you have 1 heart or 5 cards, whichever comes first.
Make a probability distribution for X = #cards you draw

What is the probability you draw a heart on the first try?

P(Heart) = 13/52 = 1/4

What is the probability you draw 3 or more cards?

This means the first two cards are not hearts

P(Non-Heart then Non-Heart) = 39/52 * 38/51 = .55882

What is the probability you have a heart by the time you are done?

This is 1 minus the probability of the complement, ie. You draw 5 non-hearts

1 – (39/52 * 38/51 * 37/50 * 36/49 * 35/48) = .77847

What is the probability you have 2 hearts by your third draw?

Because we stop after the first heart, the probability that you get two hearts is 0.

5. $P(A|B)=.35$, $P(B|A)=.42$, A and B are independent.

What is $P((A \cup B)^C)$?

Solution: A and B independent means $P(A|B) = P(A)$ and $P(B|A)=P(B)$ and $P(A \cap B)=P(A)P(B)$

So $P(A)=.35$, $P(B)=.42$ and $P(A \cap B)=.147$

$P(A \cup B)=.35+.42-.147=.623$ by the general addition rule

The probability of the complement is 1-.623=.377

6. $P(A|B) = .30$ and $P(B|A) = .30$

What does this imply? Are B and A independent? Disjoint?

This does not imply they are independent, it just means that $P(A) = P(B)$ and $P(A \cap B)/P(B) = .30$

But this DOES mean that they are not disjoint – they cannot be.

*****If A and B are not necessarily independent, give an example.**

There are many examples – for instance $P(B)=.10$, $P(A)=.10$ and $P(A \cap B)=.03$

*****If A and B are independent, what must be P(A) and P(B)?**

If A and B are independent, that means that $P(A|B)=P(A)$ so $P(A)=.30$ and $P(B)=.30$

7. You have a Die with a skull on 3 sides, and the remaining sides have a 1, 2 and 3. You roll the die 30 times.

What is the mean number of times you expect to get a skull? What is its standard deviation?

$E(X)=30*.5 = 15$, $SD(X)=\sqrt{(Var(X))} = \sqrt{(30*.5*.5)}=2.7386$

What is the mean number of times you expect to roll an odd number? What is its standard deviation?

Probability of rolling an odd number is 2/3 because #1 and #3 are odd. So $E(X) = 30*(2/3)=20$ and $SD(X)=\sqrt{(30* 2/3 * 1/3)}=2.582$

8. $P(A) = P(B)$

Is it possible for the two events to be independent? If so, give an example.

Yes – for example, $P(A)=.5$, $P(B)=.5$ and $P(A \cap B)=.25=P(A)*P(B)$

Is it possible for the two events to be disjoint? If so, give an example.

Sure – For example with a fair coin $P(H)=.5$ and $P(T)=.5$ and they are disjoint events.

9. Jerry lives near the swamp, and he is always getting mosquitoes in his apartment. On any given day he swats any number of mosquitoes, from 0 up to 15. He calculated the mean number of mosqitoes to be 4.5 with a standard deviation of 1.23

He wanted to estimate how many he'd kill over the course of a week. He figures since $E(X) = 4.5$, $E(7X) = 7*E(X) = 7*4.5 = 31.5$, and $SD(X)=1.23$, so $SD(7X) = 7*SD(X) = 7*1.23 = 8.61$

What is wrong with Jerry's reasoning/calculations?

Jerry wants to calculate $E(X_1+X_2+X_3+X_4+X_5+X_6+X_7)$, because each day is a different random variable. E(7X) assumes he gets the exact same number of mosquitoes each day, which is not correct.

10. You are taking a multiple choice test with 20 questions. Each problem has 4 choices. You didn't study at all, so you just guess on every single question.

What is the probability of getting the first two questions right?

P(First two questions correct) = .25 * .25 = .0625

What is the probability of getting exactly 5 questions right?

$$P(X=5)=\binom{20}{5}\left(\frac{1}{4}\right)^5\left(\frac{3}{4}\right)^{15}\approx .20233$$

What is the probability of getting 3 or more questions right?

$P(X\geq 3) = 1 - P(X\leq 2) = 1 - \text{binomcdf}(20, .25, 2) = .90874$

What is the probability of getting every odd question wrong?

$$P(\mathit{Odds\ wrong})=\left(\frac{1}{4}\right)^{10}\cdot\left(\frac{3}{4}\right)^{10}\approx .0000000537$$

11. In the swamp you can find bullfrogs – some have a green spot on their heads and some do not. 27% of the frogs have the green spot. Let's go frog catching!

What probability distribution is appropriate to model the chance a frog you catch will have a green spot?

Bernoulli Trial

What probability distribution is appropriate to model the expected number of green spot frogs if you catch 30 frogs? What would be its expected value and standard deviation?

X~Binomial(30, .27)

E(X)=8.1, SD(X)=2.4317

What probability distribution is appropriate to model the number of frogs you need to catch until you get a green-spot frog? What would be the expected value and standard deviation?

X~Geometric(.27)

E(X)=3.7037

SD(X)=3.1644

***** Suppose the green-spot frogs are better at hiding, and you are half as likely to catch a green-spot frog as the regular kind. If you go out and catch 30 frogs, what is the expected number of green-spot frogs you'll catch?**

Effectively you will encounter half as many green spot frogs as are proportional to the population. So if there were 100 frogs, we expect 73 to be regular and 27 to have green spots, but you'd only see 13.5 of them.

$$P(\mathit{Capture\ Grn\ Frog})=\frac{13.5}{73+13.5}=.1561$$

E(X)=30*.1561 = 4.682

12. Which are requirements for a valid probability distribution for random variable X?

(a) All probabilities are known

(b) The probabilities of all possible values of X must add up to 1

(c) The probabilities must be in ascending order

(d) Every possible value of X must have a probability between 0 and 1 (inclusive)

(e) Every value must have a positive probability (or zero)

(f) There must be a finite number of possible values for X

(g) X can only take discrete values

13. The following table gives the percentages of the population who wear glasses and own SUVs:

		Glasses	
		Wear Glasses	No Glasses
SUV	Owns an SUV	0.05	x
	Doesn't own an SUV	0.4	y

If "Owning an SUV" and "Wearing Glasses" are independent, what are the values of x and y?

Solution: the two events being independent implies that P(SUV and Glasses) = P(SUV) * P(Glasses)

P(Glasses) = .05 + .4 = .45

P(SUV) = .05 + x

.05 = (.45) * (.05 + x) = .0225 + .45x

After a little algebra, we get x= .06111

Also, 1 = .05 + .4 + .06111 + y so y = .48889