Conditional Probability Example

You are taking a bus to Cleveland, stay the night and then take another bus to New York City, and you bring a bag of trail mix with you. If the Chicago-Cleveland bus hits traffic, you'll get super hungry and there's a 90% chance you will eat the trail mix. If the bus arrives to Cleveland without hitting traffic there is only a 55% chance that you will devour the trail mix. There is a 45% chance the bus hits traffic.

So you get to Cleveland and stay the night in a hotel. The next day you have a good breakfast and take the bus to New York, stopping along the way for a meal (you don't touch the trail mix on the way to New York, whether or not you still have any)

When you get to New York, you suffer amnesia and can't remember whether or not the bus ride from Chicago to Cleveland hit traffic. All you know is that you ate all of your trail mix. What is the probability that your bus to Cleveland was delayed in traffic?

Trail Mix on the Bus

Key points:

- There is a 45% chance the bus will hit traffic
- If the bus hits traffic you eat the trail mix with 90% probability
- If there is no traffic, you eat the trail mix with 55% probability
- You know you ate the trail mix

Two events are “Traffic” and “Eat Trail Mix” - call them T and E

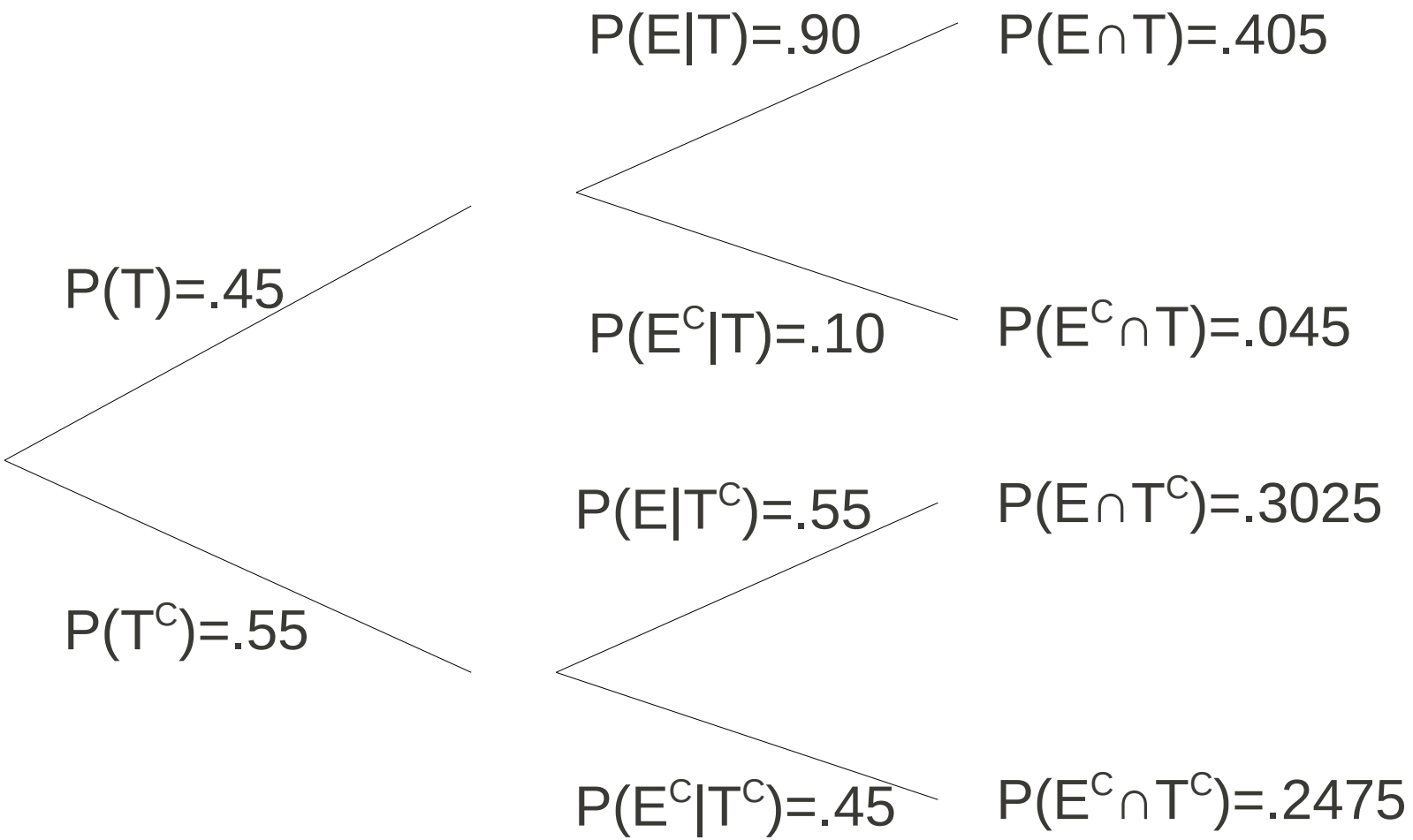
$P(T) = .45$

$P(E|T) = .90$

$P(E|T^C) = .55$

We want to know $P(T|E)$

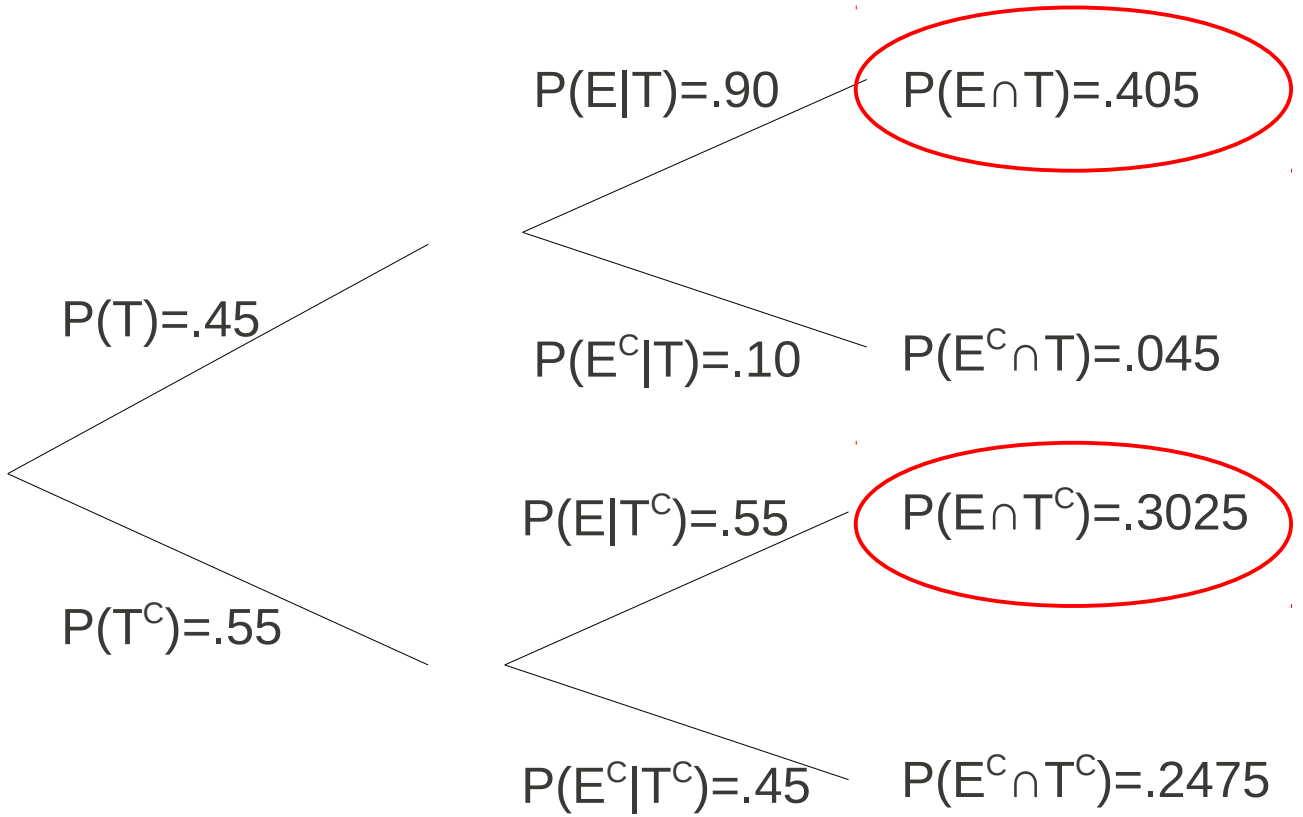
Tree diagram



Conditional Probability Calculation

The formula for conditional probability gives:

$$P(T|E)=\frac{P(T\cap E)}{P(E)}=\frac{P(T\cap E)}{P(E\cap T)+P(E\cap T^{C})}=\frac{.405}{.405+.3025}=0.5724$$



Use Bayes' Theorem

$$P(T|E) = \frac{P(E|T) \cdot P(T)}{P(E|T) \cdot P(T) + P(E|T^C) \cdot P(T^C)} = \frac{.90 \cdot .45}{.90 \cdot .45 + .55 \cdot .55}$$

$$\ldots = \frac{.405}{.405 + .3025} = 0.5724$$

Color Blind Kids

In a certain population suppose 4% of boys are color blind and 1.4% of girls are color blind. We do a color blind test to a sample of children from this population, 60% of whom are boys. What percentage of the color blind children do we expect to be girls?

Color Blind Kids, cont.

Key points:

- There is a 40% chance a tested child will be a girl
- If the tested child is a girl, she is color blind with 1.4% probability
- If the tested child is a boy, he is color blind with 4% probability

Two events are “Color Blind” and “Girl” - call them C and G

$P(G) = .40$

$P(C|G) = .014$

$P(C|G^C) = .04$

We want to know $P(G|C)$

(i.e. the probability a child is a girl given that the child is color blind)

Tree diagram