Stat 381: Applied Statistical Methods I **Spring 2015**

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Instructor: Brian Powers

3.1 Sample Spaces

Definition 3.1. An **experiment** is any process that generates a set of data.

Definition 3.2. The set of all possible outcomes of a statistical experiment is called the **sample space** and is represented by the symbol S. Each outcome of the experiment is an **element** of the sample space.

Example 3.3. Suppose you are to flip a fair coin twice, and record the flips. The sample space would be

$$S = \{HH, HT, TH, TT\}.$$

Example 3.4. Suppose you flip a fair coin twice. If both flips are heads, then you roll a 6-sided die. The sample space may be represented by the set

$$S = \{1, 2, 3, 4, 5, 6, HT, TH, TT\}.$$

Sample spaces use set notation, so to describe complicated sample spaces, we may use the **rule method**.

Example 3.5. Suppose we are to ask a person how many birthday parties they have attended. Then the sample space may be represented as

$$S = \{x | x \in \mathbb{Z} and x \geq 0\}$$

3.1.1 Events and Set Operations

Definition 3.6. An **event** is a subset of a sample space.

Example 3.7. Suppose in an experiment we are to draw a random card from a standard deck of 52 cards. The sample space is

$$S = \{2\clubsuit, \dots, A\clubsuit, \dots, 2\heartsuit, \dots, A\heartsuit\}.$$

Some events are:

$$A = \{x | x \text{ is a heart}\}$$

$$B = \{x | x \text{ is a face card}\}$$

The **empty set**, or $\emptyset$ is used to represent an event that cannot happen.

Definition 3.8. The complement of an event A is the set of all outcomes of S that do not belong to A, and is represented by A'

Definition 3.9. The **union** of two events A and B, represented by $A \cup B$ is the set of all outcomes in either A or B (or both).

Definition 3.10. The **intersection** of two events A and B, represented by $A \cap B$ is the set of all outcomes common to both A and B. If $A \cap B = \emptyset$ then we say A and B are **mutually exclusive** (or **disjoint**).

Example 3.11. Suppose you are to roll a 10 sided die once and observe the number rolled. Let $A =$"The number rolled is even" and $B =$"The number rolled is divisible by 3". We could instead write

$$A = \{2, 4, 6, 8, 10\}, B = \{3, 6, 9\}.$$

The complement events are

$$A' = \{1, 3, 5, 7, 9\}, B' = \{1, 2, 4, 5, 7, 8, 10\}.$$

The union and intersection respectively are

$$A \cup B = \{2, 3, 4, 6, 8, 9, 10\}, A \cap B = \{3\}$$

Definition 3.12. The **cardinality** of a set A is the number of elements in the set, and is represented by $n(A)$.

Definition 3.13. A sample space is **discrete** if the elements may be enumerated (it is countable). A **continuous** sample space has infinite cardinality and is uncountable.

3.1.2 Venn Diagrams

3.1.3 The Inclusion/Exclusion Principle

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$$