\( \mu = \beta \)

If \( X \sim \text{exp}(\beta) \) same as \( X \sim \text{Gamma}(1, \beta) \)

\[
f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} \quad x \geq 0
\]

The time between equipment breakdowns at a factory follows an \( \text{exp} \) with mean 20 days.

What is the prob a month goes by without a breakdown?

\( X \sim \text{exp}(20) \) \quad \text{month} = 30 \text{ days}

\[
P(X > 30) = \int_{30}^{\infty} \frac{1}{20} e^{-\frac{x}{20}} \, dx
\]

\[
= \left[ -e^{-u} \right]_{1.5}^{\infty}
\]

\[
= e^{-1.5} = 0.2231
\]

\[
1 - F(30)
\]

\[
\text{normalcdf}(a, b, \mu, \sigma)
\]

\[
P(X > x) = \text{normalcdf}(x, \infty, \mu, \sigma)
\]

\[
1 - \text{normalcdf}(\infty, x, \mu, \sigma)
\]

\[
x \sim N(\mu + 0.3)
\]
For \( X_1, \ldots, X_n \) \( \text{iid} \ N(\mu, \sigma^2) \)

\[
E(\overline{X}) = \mu \quad \text{Var}(\overline{X}) = \frac{\sigma^2}{n}
\]

\( \overline{X} \sim N(\mu, \frac{\sigma^2}{n}) \)

In General:

**Thm Central Limit Theorem**

If \( X_1, \ldots, X_n \) \( \text{iid} \) from a distribution with mean \( \mu \), and variance \( \sigma^2 \),

\[
Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}
\]

follows a \( N(0,1) \) distribution as \( n \to \infty \)

even for \( n \geq 30 \) the distribution is approximately normal

If distribution was "bellshaped" to begin with, \( n \) can be even smaller.

\( \overline{X} \ \text{approx} \ N(\mu, \frac{\sigma^2}{n}) \)
If the diameter of ball bearings follows a normal dist. with $\mu = 3$ mm, $\sigma = 0.1$ mm, what is the prob that a single ball bearing is more than 3.2 mm wide?

$$X \sim N(3, 0.1^2)$$

$$P(X > 3.2) = \Phi(3.2, 0.1)$$

$$\approx 0.02275$$

Take a sample of 10 ball bearings. What is the prob that $\bar{X} > 3.2$?

$$\bar{X} \sim N(3, \frac{0.1^2}{10})$$

$$\sigma_{\bar{X}} = \frac{0.1}{\sqrt{10}} = 0.0316$$

$$P(\bar{X} > 3.2) = \Phi(3.2, 0.0316)$$

$$\approx 0.0$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$$