Negative Binomial

Archer in Tournament.
with prob .72 of hitting the target.
If he can make 3 hits in 5 tries or fewer, he goes on to next round.

\[ P \left( \text{He goes onto next round} \right) \]

\[ X = \# \text{ attempts until 3 hits} \quad k = 3 \]

\[ P \left( X \leq 5 \right) \]

Say \[ X \sim \text{Neg Binom} \left( k, p \right) \]

\[ f(x) = P(X = x) = \binom{x-1}{k-1} p^k q^{x-k} \]

\[ \frac{\text{S}}{\text{SFFSS}} \frac{\text{S}}{\text{SSFFS}} \frac{\text{S}}{\text{SSSSS}} \]

\[ P(X \leq 5) = P(X = 3) + P(X = 4) + P(X = 5) \]

\[ \left( \frac{3}{2} \right) (0.72)^3 (0.28)^2 + \left( \frac{3}{2} \right) (0.72)^3 (0.28) + \left( \frac{4}{2} \right) (0.72)^3 (0.28)^2 \]

Throw a javelin, it falls randomly uniformly dist. from 10 yards to 13 yards away.

\[ P \left( \text{more than 12.2 yards away} \right) \]

\[ X \sim \text{Unif} \left( 10, 13 \right) \quad P(X \geq 12.2) \]
\( X \sim N(\mu, \sigma^2) \)

\[
P(X \leq x_1) = \int_{-\infty}^{\frac{x_1 - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2} \, dx
\]

Use \( z = \frac{x - \mu}{\sigma} \)  \( \, dz = \frac{dx}{\sigma} \)

\[
= \int_{-\infty}^{\frac{x_1 - \mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} \, dz
\]

\[= P(z \leq z_1) = \Phi(z_1)\]

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Find \( x \) s.t. \( P(X \leq x) = .45 \)

\( X \sim N(40, 6^2) \)

\( P(Z \leq z) = .45 \)

\[z = \Phi^{-1}(.45)\]

\[z = -1.2566\]

\[
z = \frac{x - \mu}{\sigma} \quad \Rightarrow \quad z = \left| \frac{x - 40}{6} \right| = -1.2566
\]

\[x = 6(-1.2566) + 40 = 39.24\]
What is \( P(-1 < Z < 1) \)?

\[
\text{normalcdf}(-1, 1) = 0.6826
\]

\[
P(Z < -1) = 0.1587
\]

\[
P(-2 < Z < 2) = 0.9545
\]

\[
P(-3 < Z < 3) = 0.9973
\]

\[
\begin{align*}
\mu - 3\sigma & \quad \mu - 2\sigma & \quad \mu - \sigma & \quad \mu & \quad \mu + \sigma & \quad \mu + 2\sigma & \quad \mu + 3\sigma \\
68\% & \quad 95\% & \quad 99.7\%
\end{align*}
\]

\[
X \sim N(3, 2^2)
\]

\[
P(X > 5)
\]

\[
X \sim \text{Poisson}(\lambda)
\]

\[
X \sim \text{Gamma}(\alpha, \beta)
\]

\[
\lambda t
\]
\[ \int_{12.2}^{13} \frac{1}{13-10} \, dx = \frac{1}{3} \left[ 13 - 12.2 \right] = \frac{8}{3} = \frac{4}{15} \]

\[ P(x \geq 12.2) = \frac{13 - 12.2}{13 - 10} \]

Roulette table: If cost is $1 and play Red

1-36 odds Red

00 green

If win, get $2.

What is the expected value?

\[ P(\text{roll red}) = \frac{18}{38} = \frac{9}{19} \]

Winning are:

\[ X = \begin{cases} 5 \text{ win (Net)} & \text{if roll red} \\ -1 & \text{if not red} \end{cases} \]

\[ E(X) = \frac{10}{19} + 1 \left( \frac{9}{19} \right) = -0.0526 \]