3/2/2015

Negative Binomial

Archer in Tournament.
with prob .72 of hitting the target.

If he can make <u>3</u> hits in 5 tries or fever,
he goes on to next round.

P(He goes onto next round)

X = # attempts until 3 hits

$P(X \le 5)$

$k=3$
$p=.72$

Say $X \sim \text{Neg Binom}(k, p)$

$f(x) = P(X=x) = \binom{x-1}{k-1} p^k q^{x-k}$

$\underbrace{\_\ \_\ \_\ \_}_{x-1}\ \underline{S}$

k-1 successes
$x-k$ failures

FFSSS
SSFFS
~~SSSFF~~

$P(X\le5) = P(X=3)+P(X=4)+P(X=5)$

$\binom{2}{2}(.72)^3(\cancel{.28})^0 + \binom{3}{2}.72^3(.28)^1 + \binom{4}{2}(.72)^3(.28)^2$

Throw a javelin it falls randomly uniformly dist.
from 10 yards to 13 yards away.

P(more than 12.2 yards away)

$X \sim \text{Unif}_c(10,13) \quad P(X \ge 12.2)$

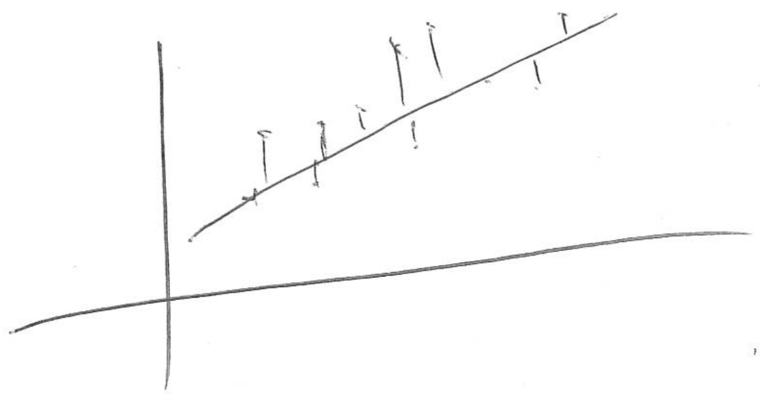
$X \sim N(\mu, \sigma^2)$

$$P(X \le x_1) = \int_{-\infty}^{x_1} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}\,dx$$

use $z = \frac{x-\mu}{\sigma}$ $\quad dz = \frac{dx}{\sigma}$

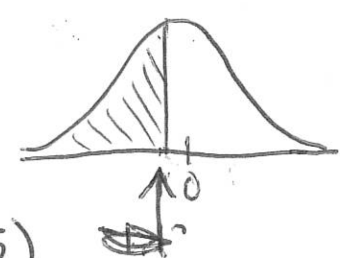
$$= \int_{-\infty}^{\frac{x_1-\mu}{\sigma}=z_1} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}\,dz$$

$$= P(Z \le z_1) = \Phi(z_1)$$



Find x s.t. $P(X \le x) = .45$

$X \sim N(40, 6^2)$ $\quad P(Z \le z) = .45$



$z = \Phi^{-1}(.45)$

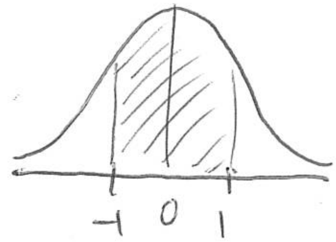
$z = -.12566$

$z = \frac{x-\mu}{\sigma}$ $\quad z = \boxed{\frac{x-40}{6} = -.12566}$

$x = 6(-.12566) + 40$

$= 39.24$

What is $P(-1 < Z < 1)$

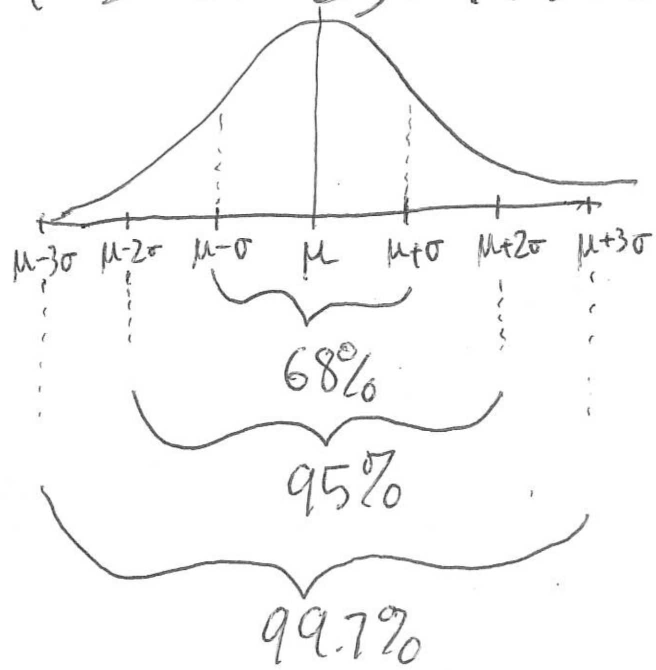


normalcdf$(-1, 1) = .6826$

$P(Z < -1) = .1587$

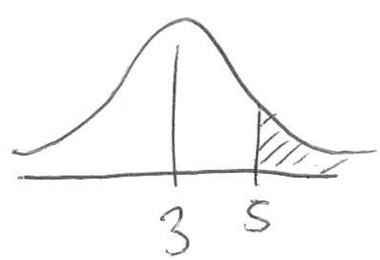
$P(-2 < Z < 2) = .9545$

$P(-3 < Z < 3) = .9973$

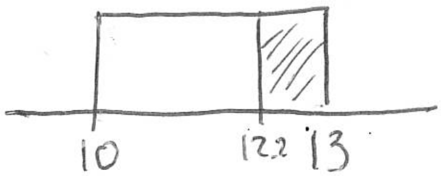


1 $X \sim N(3, 2^2)$

$P(X > 5)$



$X \sim \text{Poisson}(\lambda t)$ / $X \sim \text{Gamma}(\underset{\underset{10}{\uparrow}}{\alpha}, \underset{\underset{\frac{1}{\lambda t}}{\uparrow}}{\beta})$

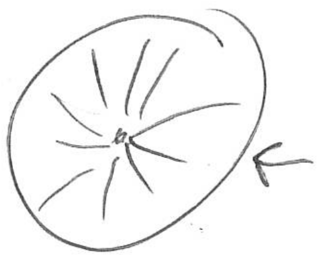


$$\int_{12.2}^{13} \frac{1}{13-10}\, dx$$

$$= \frac{1}{3}\left[13-12.2\right] = \frac{.8}{3} = \frac{4}{15}$$

$$P(X \geq 12.2) = \frac{13-12.2}{13-10}$$

Roulette table If cost is \$1 and play Red



#1-36 odds Red
evens Black

0
00 ← green

38

if win get \$2.

What is the expected value?

$$P(\text{roll red}) = \frac{18}{38} = \frac{9}{19}$$

winnings are

$X = \$$ I win (Net)

$$X = \begin{cases} -1 & \text{if not red} \\ 1 & \text{if red.} \end{cases}$$

$$E(X) = -1\left(\frac{10}{19}\right) + 1\left(\frac{9}{19}\right) = -.0526$$

X can take values
$a, a+1, \ldots\ldots, b$

$$f(x) = \frac{1}{b-a+1}$$