3/20 Stat 381

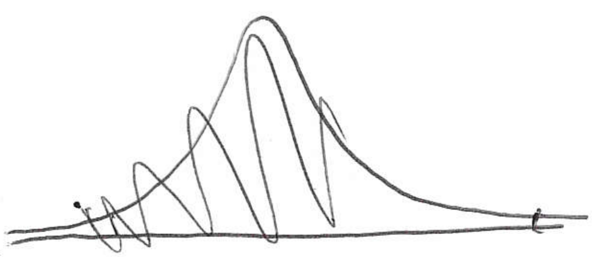
# T-distribution

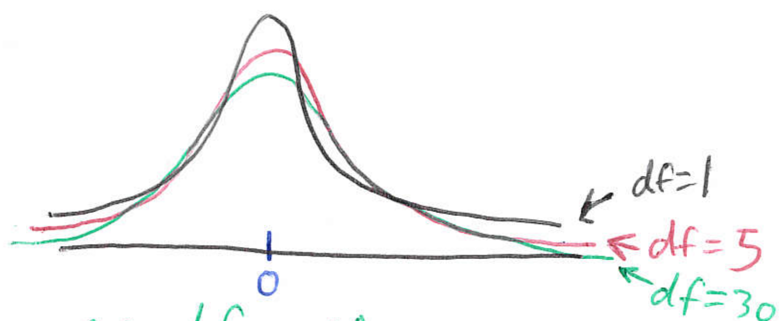
If $X_1, \dots, X_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1) \leftarrow \text{CLT.}$$

If $\sigma$ is <u>unknown</u> use "$S$" sample std. dev.

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}} \sim \text{T-distribution with } n-1 \text{ degrees of freedom}$$






as $df \to \infty$

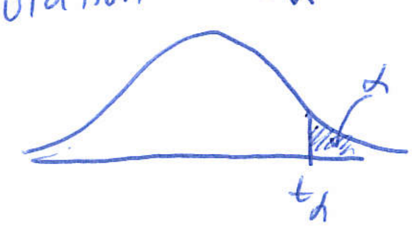
T-dist $\to N(0,1)$

ex Say $T$ follows a t-distribution with 7 df.,

$P(T > 1.3) =$ between .15 and .10

tcdf(lower, upper, df) $\searrow$ .11738

notation $t_\alpha$ as $P(T > t_\alpha) = \alpha$




ex for 7 df,

$t_{.10} = 1.415$

tinv$(1-\alpha, df)$

We will use a T-statistic $T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$

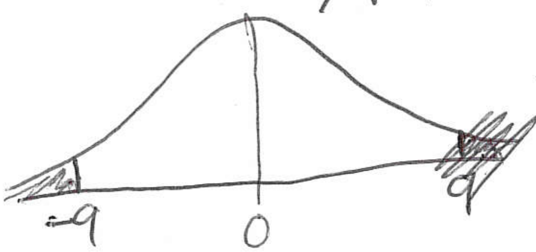
if
- $\sigma$ is unknown
- $n$ is small (rule of thumb $< 30$)

ex suppose mean mpg for Toyota Camry is 32 mpg we sample 7 cars from a mfg plant and calculate $\bar{x} = 30.3$ $s = .5$

How unusual is our observation if $\mu = 32$?

$$t = \frac{30.3 - 32}{.5/\sqrt{7}} = -8.99 \approx -9$$

$df = 6$




~~$2 \cdot P(T > 9)$~~

$P(T < -9) = .0000526$

know $\chi^2(v) \equiv \text{Gamma}\left(\frac{v}{2}, 2\right)$

$\exp(\beta) \equiv \text{Gamma}(1, \beta)$

what precise dist. can be used to approx Gamma$(50, 3)$

$X_1 + \cdots + X_{50}$ indep $X_i \sim \exp(3)$

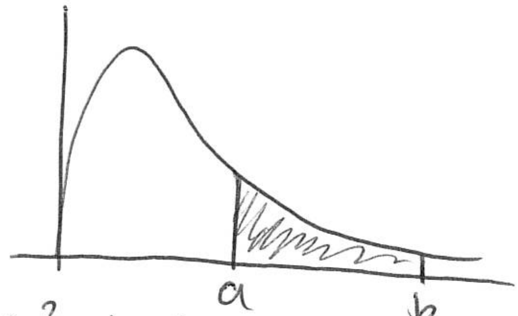
$\mu = \alpha\beta = 150$

$\sigma^2 = \alpha\beta^2 = 450$

$E(X_i) = 3$

$Var(X_i) = 9$

$$S_{50} \overset{\text{approx}}{\sim} N(\underbrace{50 \cdot 3}_{\mu}, \underbrace{50 \cdot 9}_{\sigma^2})$$




$\chi^2 cdf(a, b, v = df)$

ex $\chi^2$ with 18 df $(\chi^2(18))$

$P(10 < X^2 < 20)$

$= \chi^2 cdf(10, 20, 18)$

if $X \sim Binom(n, p)$

$f(x) = \binom{n}{x} p^x (1-p)^{n-x}$

Bern. $X = \begin{cases} 1 & \text{with prob } p \\ 0 & \text{w. p. } 1-p = q \end{cases}$ $E(X) = p$

$Var(X) = pq$

If $X_1, \dots, X_n$ iid Bern$(p)$

$$\sum X_i = Y \sim Bin(n, p) \qquad E(Y) = np \quad Var(Y) = npq$$

↳ Geometric How many trials until the first success!

↳ Negative Binomial How many trials until $k$ successes

↳ Hypergeometric Marbles from a jar

Poisson # occurrences in $t$ units of time where $\lambda$ avg in 1 unit of time.

$$X \sim Poisson(\lambda t) \qquad E(X) = \lambda t \quad Var(X) = \lambda t$$

$$f(x) = \frac{(\lambda t)^x e^{-\lambda t}}{x!}$$

Gamma$(\alpha, \beta)$ $\alpha \equiv$ # occurrences we wait for

$\beta = \frac{1}{\lambda t}$