\[ X \sim \text{Unif}_c(1, 5) \]

\[ X \geq 2.5 \text{ and } x \leq 4 \]

\[
P(X > 2.5 \mid x \leq 4) = \frac{P(2.5 < X \leq 4)}{P(x \leq 4)} = \frac{\frac{3}{4}}{\frac{3}{4}}
\]

\[ P(1 \leq x \leq 4) = \frac{1.5}{3} = \frac{1}{2} \]

\[ Y \sim \text{Unif}_c(1, 4) \]

\[ P(Y > 2.5) = \frac{1}{2} \]

\[ P(X > 3.5 \mid x \leq 4) = \frac{P(3.5 < x \leq 4)}{P(x < 4)} = \frac{\frac{5}{4}}{\frac{3}{4}} = \frac{\frac{5}{3}}{\frac{1}{3}} = \frac{1}{6} \]

---

Say waiting time at a DMV line follows an exponential distribution with mean waiting time of 6 minutes.

What is the probability that it takes fewer than 2 minutes to help a customer?

\[ X \sim \exp(6) \quad f(x) = \begin{cases} \frac{1}{6} e^{-\frac{x}{6}} & x \geq 0 \\ 0 & x < 0 \end{cases} \]

\[ \int_{0}^{2} \frac{1}{6} e^{-\frac{x}{6}} dx = \left[ -e^{-\frac{x}{6}} \right]_{0}^{2} = -e^{-\frac{1}{3}} + 1 \]
Let \( u = \frac{x}{6} \)
\( du = \frac{dx}{6} \)
\[
\int_0^1 e^{-x/6} dx = \int_0^{1/3} e^{-u} du = -e^{-u/3} \bigg|_0^{1/3} = -e^{-1/3} + 1 = 0.2835
\]

**Gamma \((\alpha, \beta)\)**

The amount of time it takes for 5 customers follows a Gamma distribution with \( \alpha = 5 \) and \( \beta = 6 \)

5 people are in front of you,

What is the probability you'll be at the counter in less than 30 min?

\[ X \sim \text{Gamma} \left( \frac{\alpha}{\beta}, \frac{1}{\beta} \right) \]

\[
P(X < 30) = \int_0^{30} \frac{1}{6^5 \Gamma(5)} x^{\frac{\alpha}{\beta} - 1} e^{-\frac{x}{\beta}} dx
\]

\[ u = \frac{x}{6} \quad du = \frac{dx}{6} \]

\[
= \int_0^5 \frac{1}{\Gamma(5)} (4-u) u e^{4-u} du
\]

\[ F(5; 5) = 0.5595 \]

1 - Poisson CDF(5, 4)

\[ x \leftarrow x - 1 \]
If \( X_1, X_2 \) independent,
\[ X_i \sim \text{Gamma}(\alpha_i, \beta) \]
\[ X_1 + X_2 \sim \text{Gamma}(\alpha_1 + \alpha_2, \beta) \]
say \( X_1, \ldots, X_n \) indp. exp(\( \beta \)) \( \equiv \text{Gamma}(1, \beta) \)
\[ \sum_{i=1}^{n} X_i \sim \text{Gamma}(n, \beta) \]

If \( X_1, X_2 \) are Normal, independent
\[ X_i \sim N(\mu_i, \sigma_i^2) \]
\[ X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2) \]
\[ \Rightarrow X_1, \ldots, X_n \text{ indp. Normal } X_i \sim N(\mu_i, \sigma_i^2) \]
\[ \sum X_i \sim N(\sum \mu_i, \sum \sigma_i^2) \]
Ch 8 Sampling Distributions

Sampling from population

Population $\leftrightarrow$ Probability Distribution.

Sample size $n$ - each observation is independent (i.e., random) and all from same population,

we say sample is iid independent and identically distributed

A statistic is any function of random sample data. 

- Sample mean $\bar{X} = \frac{X_1 + \ldots + X_n}{n}$

- Sample median
  - $\tilde{X} = \left\{ \begin{array}{ll} X_{(n+1)/2} & \text{n odd} \\ \frac{X_{(n/2)} + X_{(n/2+1)}}{2} & \text{n even} \end{array} \right.$

- Sample mode value most common
  - eg $E(X_i) = \mu$

Take $X_1, \ldots, X_n$ iid distribution with mean $\mu$ variance $\sigma^2$

$E(\bar{X}) = E\left(\frac{1}{n}(X_1 + \ldots + X_n)\right) = \frac{1}{n} E(X_1 + \ldots + X_n) = \frac{1}{n} \sum_{i=1}^{n} E(X_i) = \frac{1}{n} \sum_{i=1}^{n} \mu = \frac{1}{n} n \mu = \mu$
\[ E(\bar{X}) = \mu \quad \text{regardless of type of distribution} \]

\[ \text{Var}(\bar{X}) = \text{Var} \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \text{Var} \left( \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \sum_{i=1}^{n} \text{Var}(X_i) \]

\[ = \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{1}{n^2} n \cdot \sigma^2 = \frac{\sigma^2}{n} \]