Confidence Intervals

We use $\bar{x}$ to estimate $\mu$. Say we want to estimate an interval $(LB, UB)$ which we are $100(1-\alpha)\%$ confident contains $\mu$. E.g., $95\%$

By CLT, we know that $\Rightarrow \alpha = .05$

$\bar{x}$ is approx. normal with $\mu_{\bar{x}} = \mu$

We want to find $\frac{3\alpha}{2}$

Such that $P\left(-\frac{3\alpha}{2} < Z < \frac{3\alpha}{2}\right) = 1-\alpha$

By CLT $\Rightarrow P\left(-\frac{3\alpha}{2} < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < \frac{3\alpha}{2}\right) = 1-\alpha$

$\Rightarrow P\left(-\frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}} < \bar{x} - \mu < \frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

$\Rightarrow P\left(-\bar{x} - \frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}} < -\mu < -\bar{x} + \frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

$\Rightarrow P\left(\bar{x} - \frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}} > \mu > \bar{x} - \frac{3\alpha}{2} \frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$
If $\sigma$ is known and $n$ is big enough for us to use CLT, a 100(1-$\alpha$)% CI for $\mu$ is:

$$\left( \bar{x} - 3\frac{\sigma}{\sqrt{n}}, \bar{x} + 3\frac{\sigma}{\sqrt{n}} \right)$$

Example: Want a 92% CI for weight of frogs in a certain lake. Random sample of 65 frogs, calculate $\bar{x} = 22$ oz. Say we know $\sigma = 5$ oz.

$$22\text{oz} \pm 1.75 \frac{5}{\sqrt{65}}$$

22 oz $\pm$ 1.0853

(20.915, 23.0853)