Stat 381 4/6

Confidence Intervals

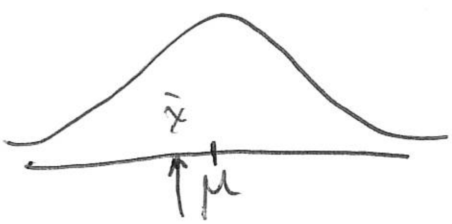
we use $\bar{x}$ to estimate μ

Say we want to estimate an interval (LB, UB) which we are $100(1-\alpha)\%$ confident contains μ eg 95%

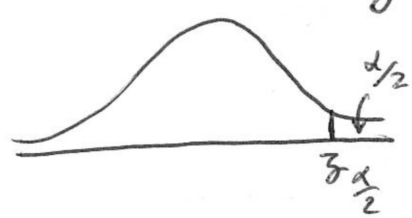
$95 = 100(1-\alpha)$

By CLT we know that $\Rightarrow \alpha = .05$

$\bar{X}$ is approx normal with $\mu_{\bar{X}} = \mu$



We want to find $z_{\frac{\alpha}{2}}$



such that $P\left(-z_{\frac{\alpha}{2}} < Z < z_{\frac{\alpha}{2}}\right) = 1-\alpha$

by CLT $\Rightarrow P\left(-z_{\frac{\alpha}{2}} < \dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} < z_{\frac{\alpha}{2}}\right) = 1-\alpha$

$\Rightarrow P\left(-z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < \bar{X}-\mu < z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

$\Rightarrow P\left(-\bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} < -\mu < -\bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

$\Rightarrow P\left(\bar{X} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} > \mu > \bar{X} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) = 1-\alpha$

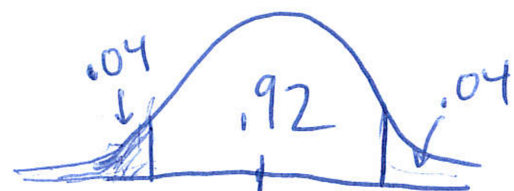
If σ is known and n is big enough for us to use CLT

a $100(1-\alpha)\%$ CI for μ is

$$\left(\bar{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}},\ \bar{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right)$$

ex) Want a 92% CI for weight of frogs in a certain lake. random sample of 65 frogs, calculate $\bar{x} = 22$ oz

Say we know $\sigma = 5$ oz



invNorm(.04) = -1.75

$$22\text{oz} \pm 1.75\frac{5}{\sqrt{65}}$$

$$22\text{oz} \pm 1.0853$$

$$(20.915,\ 23.0853)$$