

## Chapter 2

1. In how many ways can 5 different trees be planted in a row?
2. How many subsets of  $S = \{1, 2, 3, \dots, 100\}$  contain 2 elements?
3. In how many ways can 100 students be assigned 40 into dorm A, 35 into dorm B and 25 into dorm C?
4. How many ways can a president, a vice president and a 3 person advisory committee be assigned from 30 people?

## Chapter 3

1. If  $A$  and  $B$  are independent events with  $P(A) = 0.6$  and  $P(B) = 0.3$ , find the following:  
 (a)  $P(A \cup B)$    (b)  $P(A' \cap B)$    (c)  $P(A' \cup B')$    (d)  $P(A|B)$    (e)  $P(B'|A')$
2. If  $A$  and  $B$  are mutually exclusive events with  $P(A) = 0.6$  and  $P(B) = 0.3$ , find the following:  
 (a)  $P(A \cup B)$    (b)  $P(A' \cap B)$    (c)  $P(A' \cup B')$    (d)  $P(A|B)$    (e)  $P(B'|A')$
3. You have to pass through an obstacle course. The probabilities that you make a mistake on each of the 4 obstacles is (respectively) 0.2, 0.3, 0.25 and 0.5. You pass the course if you make no more than 2 mistakes. What is the probability that you pass the course?
4. A random person has a probability of 0.36 of being a descendant of Ghengis Khan. A company advertises a blood test which can tell you if you are a descendant, and it is correct 99% of the time. If you take the test and it comes back negative, what is the probability you actually ARE descended from Ghengis?
5. Your new neighbors have 2 children and you know at least one of them is a boy. You see one of them playing in the backyard and he is a boy - what is the probability the other child is a boy too? (Assume boys and girls are born with equal probability).
6. My fear of animals depends on how many legs they have, given in the table below, along with the probability that a random encounter will have the given number of legs:

| # Legs                   | 0    | 2    | 4    | 6    | 8    | > 8  |
|--------------------------|------|------|------|------|------|------|
| Probability of encounter | 0.05 | 0.70 | 0.10 | 0.05 | 0.09 | 0.01 |
| Probability of fear      | 0.5  | 0.1  | 0.3  | 0.6  | 0.8  | 0.9  |

If I encounter an animal and I am afraid of it, what is the probability that it had no legs?

## Chapter 4

1. A jar has 5 red and 10 blue marbles. I pick a handful of 4 marbles out. Let  $X$  be the number of red marbles in my hand. Find the pmf of  $X$ , find its expected value and its variance.
2. A 65 year old couple are considering a joint life insurance policy. The man has a probability of .90 of living at least 5 more years, .95 for the woman (and assume the event of either person dying is independent of the other). The insurance polity pays \$100,000 if one of them dies and \$150,000 if both die during this time. What is a fair cost for this policy?
3. **The St. Petersburg Paradox** The game is as follows: You pay \$1000 to play. The pot starts at \$1. I flip a coin. If it is a tails you take the pot and the game is over. If it is heads then I double the pot. What is the (net) expected value of the game? What if it costs \$1,000,000 to play?
4. Consider a random variable  $X$  which can take any positive integer value (i.e.  $1, 2, 3, \dots$ ). Its pmf is

$$f(x) = \frac{c}{4^x}.$$

Find the value  $c$ , find its cdf, and calculate  $P(X < 4)$ . Try to find its expected value (tricky).

5. Determine  $k$  so that the following is a valid pdf for  $X$ :

$$f(x) = k/\sqrt[3]{x}, \quad 0 < x < 4$$

Then find  $E(X)$ ,  $P(X < 2)$ , and  $Var(X)$ .

6. The cdf of  $Y$  is given by

$$F(y) = \begin{cases} 0 & y < 1 \\ \ln(y) & 1 \leq y \leq e \\ 1 & x > e \end{cases}$$

Find  $f(y)$ ,  $E(Y)$  and  $Var(Y)$ . Find  $P(Y > 2)$ .

7. If  $X$  and  $Y$  are independent, with  $\mu_X = 10$ ,  $\sigma_X^2 = 8$ ,  $\mu_Y = -3$ ,  $\sigma_Y = 2$ , find the mean and variance of:  
(a)  $W = 3X - 8Y$  (b)  $T = X + Y$
8. Let  $X$  be the number of heads out of 4 flips of a fair coin, let  $Y_i$  be the  $i$ th roll of a 6-sided die. Find the mean and variance of:  
(a)  $X + Y_1$  (b)  $Y_1 - Y_2$  (c)  $2X + 3Y_2 + 1.52$