Final Exam Review

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The final exam will be approximately 1/3 on chapters 1-6 and 2/3 on chapters 8-10. You will be allowed to bring a calculator (statistical functions allowed) and a 2-sided sheet with formulas and notes.

1   Topics summary

1.1   Chapter 2: Introduction to Statistics and Data Analysis

1. Measures of location: mean, median, mode

2. Measures of variability: range, IQR, variance, standard deviation.

1.2   Chapter 2: Probability

1. Definitions: Experiment, sample space, outcome, event

2. Use of set notation (e.g. $\cap, \cup$)

3. Counting techniques: permutations, combinations

4. Definitions: mutually exclusive events, conditional probability, independent events

5. Probability formulas: Addition Rule (for union of events), Multiplication rule (for intersection of events), Conditional Probability rule, Bayes rule

1.3   Chapter 3: Random Variables and Probability Distributions

1. Definitions: random variable, discrete, continuous, pmf/pdf, cdf

2. calculating probabilites for discrete random variables from a pmf

3. calculating probabilities for a continuous random variable from a pdf

4. calculating conditional probabilities from pdf/pmf

5. derive pmf/pdf from cdf and vice versa

1.4   Chapter 4: Mathematical Expectation

1. Definitions: mean, variance of a random variable

2. calculating mean and variance from a pmf/pdf

3. calculate expected value of a function of a random variable

4. Formulas for mean and variance of a linear combination of random variables

1.5 Chapter 5: Some Discrete Probability Distributions

1. Bernoulli, Binomial, Geometric, Poisson

2. Mean, Variance and pmf of these four distributions

3. Calculating probabilities of these four distributions

1.6 Chapter 6: Some Continuous Probability Distributions

1. Uniform, Normal, Gamma, Exponential, Chi-Squared

2. mean, variance of these distributions

3. pdf of the exponential distribution

4. calculating probabilities for Normal, Exponential and Chi-Squared distributions (not Gamma)

5. Understanding a Poisson Process - how to transform λ from a Poisson distribution to β in an exponential/Gamma distribution

1.7 Chapter 8: Fundamental Sampling Distributions and Data Descriptions

1. Concept of a "sampling distribution"

2. Central Limit Theorem

3. t-distribution

4. Linear combination of independent random variables from the same family (Poisson, Bernoulli, Binomial, Normal, Gamma, Exponential, Chi-Squared), and how/when the distribution remains in that family

5. Sampling distribution of S^2

1.8 Chapter 9: One- and Two-Sample Estimation Problems

1. Definitions: estimator, unbiased, best unbiased estimator, standard error of an estimator, margin of error, confidence level

2. Constructing Confidence Intervals for:

   (a) Single Population Mean (σ known; σ unknown)
   (b) Difference between two Means (σs known; σs unknown but assumed equal)
   (c) Paired Observations
   (d) Single Population Proportion
   (e) Difference between two population proportions

3. Confidence interval for variance

4. Determining sample size needed to achieve a certain margin of error

1.9 Chapter 10: One- and Two-Sample Tests of Hypotheses

1. Definitions: null hypothesis, alternative hypothesis, type I error, type II error, significance level/α, β, power, simple hypothesis, one-sided hypothesis, two-sided hypothesis, critical value, critical region/rejection region, P-value

2. Hypothesis tests for

   (a) Single Population Mean (σ known; σ unknown)

   (b) Difference between two Means (σs known; σs unknown but assumed equal)

   (c) Paired Observations

   (d) Single Population Proportion

   (e) Difference between two population proportions

2 Review Problems from Ch 1-6

These represent the difficult level you may see on the exam, although the questions would be shorter and not as many.

1. Find the mean, median, mode, range, min, max, Q_1, Q_3, IQR, variance and standard deviation for the following sample data (by hand or by calculator):

$$3, 5, 5, 6, 7, 8, 10, 12, 12.5, 15, 15, 15, 16$$

2. An electrical current has to pass through 3 checkpoints: at the first there are 4 gates, at the second there are 2 gates, and at the third there are 5 gates. How many paths are there through the circuit?

3. A friend is letting you borrow 3 movies, and he has 23 to choose from. How many selections are possible?

4. Your extended family is cramming into a bus. There are 50 seats on the bus and your family has 37 members. How many ways can the 37 of you be seated on the bus (take note of the empty seats)?

5. Of the 27 students in class, 13 of them have studied. If I randomly call 3 names, what is the probability that at least one of them studied? If I call on 3 names, what is the probability that all 3 have studied given that at least one of them has studied?

6. Your aunt told you that there was a 30% chance she would vacation in Rome, otherwise she'd go to Madrid. There's a 10% chance she'd be robbed in Rome and a 20% chance she'd be robbed in Madrid. If you find out she got robbed, what is the probability it happened in Rome?

7. $P(A) = .5, P(B) = .4$ and the two events are independent. What is $P(A \cup B')$?

8. Find the value of a which makes the following a valid pmf. Then find its cdf, $P(X > 4), E(X), E(X + 4), Var(X), E(ln(X))$.

$$\begin{array}{c|ccccc} x & 1 & 2 & 3 & 4 & 5 \\ \hline P(x) & .21 & .04 & .12 & a & .33 \end{array}$$

9. find the value of c which makes the following a valid pdf for random variable X. Then find its cdf, $P(X > 4)$, $E(X)$, $E(X + 4)$, $Var(X)$

$$f(x) = \begin{cases} cx^{1/3} & 3 \le x \le 6 \\ 0 & \text{otherwise} \end{cases}$$

10. For independent random variables X and Y, $E(X) = 4, Var(X) = 13, E(Y) = 2, Var(Y) = 10$. Find $E(X - Y), Var(X - Y), E(2X + 3Y), Var(2X + 3Y)$

11. If 35% of monarch butterfly caterpillars have spots, and a random sample of 16 are captured, what is the probability of getting at least 5 with spots? What is the expected number of caterpillars with spots, and its standard deviation? If you are going to keep capturing caterpillars until you get one with spots, what is the probability you will need *at least* 3 caterpillars? What is the expected number of caterpillars you'll need to catch, and what is the standard deviation?

12. If the average number of car alarms during 4 hours is 1 in your neighborhood, and it is a Poisson process, what is the probability that you can get 8 hours of sleep without a car alarm going off? What is the expected number of car alarms during 24 hours?

13. If you arrive at work between 8:45 and 9:25 with uniform likelihood, what is the probability you will get in before 9am? What is the probability that happens at least 3 out of 5 days in a week (assuming different days' arrival times are independent)? What is your average arrival time, and what is the standard deviation?

14. If the cost of gas at a Chicago gas station is approximately normally distributed with a mean price of \$2.789 and a standard deviation of \$0.15, what is the probability that the next gas station you see will have a price above \$3.00 per gallon?

15. You're busily working through a test. If the length of time to complete each question follows an exponential distribution with mean 1.3 minutes, what is the probability that a single problem takes you more than 4 minutes to complete? what is the expected number of problems you could complete in 50 minutes?

3 Review Problems from Ch 8-10

1. If the probability that a random voter approves the governor is .45:

   (a) Suppose a random sampling of 60 voters is polled. Let X be the precise number of voters who support the governor. What distribution does X follow?

   (b) The Central Limit Theorem may be used to approximate the distribution of X by what distribution?

2. A sampling of 15 lab grown diamonds has a mean clarity of 6.43 with standard deviation .22.

   (a) Give a 95% confidence interval for the mean clarity of this lab procedure's diamonds.

   (b) Test with 5% significance whether the mean clarity is equal to 6.5 or not. Give the p-value.

   (c) Test with 5% significance whether the mean clarity is at least 6.5 or not. Give the p-value.

Also see the problems on Lia Liu's website: http://tigger.uic.edu/~yliu/