Problem 1. Random variable $X$ can take values 0, 1, or 2 with positive probability. It’s pmf is $f(x) = c(3 + x)$.

a. (3 points) Find the constant $c$.

\[
LHS = c(3+0) + c(3+1) + c(3+2) = c(12) = 1 \quad \Rightarrow \quad c = \frac{1}{12}
\]

b. (2 points) Find $P(-1 \leq X \leq 1)$.

\[
P(-1 \leq X \leq 1) = P(X = 0 \text{ or } X = 1) = f(0) + f(1) = \frac{1}{12}(3) + \frac{1}{12}(4) = \frac{7}{12}
\]

Problem 2. Random variable $Y$ has support $[0, 3]$. Its pdf is $f(y) = k(3 - y)$.

a. (3 points) Find the constant $k$.

\[
\int_0^3 k(3-y) \, dy = k \left[ 3y - \frac{1}{2}y^2 \right]_0^3 = k \left[ 9 - \frac{9}{2} \right] = \frac{9}{2}k = 1 \quad \Rightarrow \quad k = \frac{2}{9}
\]

b. (2 points) Find $P(-1 \leq Y \leq 1)$.

\[
P(-1 \leq Y \leq 1) = P(0 \leq Y \leq 1) = \int_0^1 \frac{2}{9}(3-y) \, dy = \frac{2}{9} \left[ 3y - \frac{1}{2}y^2 \right]_0^1 = \frac{2}{9} \left[ \frac{5}{2} \right] = \frac{5}{9}
\]