2. To determine the $\gcd(2^4 \cdot 3^2 \cdot 5 \cdot 7^2), 2 \cdot 3^3 \cdot 7 \cdot 11)$ take the **lowest** power of each of the **common** factors: $2, 3^2, 7$ and multiply them.

The answer is: $2 \cdot 3^2 \cdot 7$.

To determine the lcm, take the **highest** power of **each** factor: $2^3, 3^3, 5, 7, 11$, and multiply them.
Then answer is $2^3 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11$.

9. If a and b are integers and $n \geq 0$ prove that $a \bmod n = b \bmod n$ **if and only if** n divides $a - b$.

Proof: Assume that $a \bmod n = b \bmod n$.

This is equivalent to saying that $(a - b) \bmod n = 0$. By definition, we can write $a = nq_1 + r_1$ and $b = nq_2 + r_2$. But by assumption,

$$(a - b) = n(q_1 - q_2) + (r_1 - r_2) \equiv 0 \bmod n.$$

So $r_1 - r_2 = 0$ and n divides $a - b$.

Conversely, Assume that n divides $a - b$. By definition of "divides", we know that $a - b = nq$ for some integer q. But this is equivalent to saying that $(a - b) \equiv 0 \bmod n$, which is equivalent to saying that $a \bmod n = b \bmod n$.

Therefore, $a \bmod n = b \bmod n$ if and only if n divides $a - b$.

11. Let n be a fixed positive integer greater than 1. If a mod $n = a'$ and b mod $n = b'$, prove that $(a + b)$ mod $n = a' + b'$ and (ab) mod $n = (a'b')$.

Proof:
Given that a mod $n = a'$ and b mod $n = b'$, we know that $a = nq_1 + a'$ and $b = nq_2 + b'$ for some integers q_1, q_2. Then, $a + b = n(q_1 + q_2) + (a' + b')$.

Similarly,

$$ab = (nq_1 + a')(nq_2 + b') = n^2 q_1 q_2 + nq_1 b' + nq_2 a' + a'b' = n(nq_1 q_2 + q_1 b' + q_2 a') + a'b'.$$

14. Show that $5n + 3$ and $7n + 4$ are relatively prime for all n.
We apply Theorem 0.2, and the Euclidean Algorithm to show that

$$\gcd(5n + 3, 7n + 4) = 1.$$

Follow the example on page 7.

Given that : $7n + 4 = 1(5n + 3) + (2n + 1)$;
$5n + 3 = 2(2n + 1) + (n + 1)$;
$2n + 1 = 2(n + 1) - 1$

Now, $2n + 1 =$
$(7n + 4) + (5n + 3)(-1)$;
$n + 1 =$
$(5n + 3) + (2n + 1)(-2) =$
$(5n + 3) + [(7n + 4) + (5n + 3)(-1)](-2) =$
$(5n + 3) + (-2)(7n + 4) + (2)(5n + 3) =$
$(3)(5n + 3) + (-2)(7n + 4)$;
$-1 = (2n + 1) + (-2)(n + 1) =$
$(7n + 4) + (-1)(5n + 3) + (-2)((3)(5n + 3) + (-2)(7n + 4)) =$
$(5)(7n + 4) + (-7)(5n + 3)$

So $1 = (-5)(7n + 4) + (7)(5n + 3)$.