Math 320, Linear Algebra, Exam 1
Wednesday, October 1, 2008

YOU MUST SHOW ALL DETAILS OF YOUR ARGUMENTS IN THE EXAM BOOKLET TO RECEIVE CREDIT

1. Let V be a vector space and $v_i \in V$, $1 \leq i \leq n$. Give definition of $\{v_1, \ldots, v_n\}$ linearly dependent.

2. Define pivoting and prove that it preserves the solutions of a system of linear equations

$$\begin{aligned} a_{11}x_1 + \ldots + a_{1n}x_n &= b_1, \\ &\vdots \\ a_{m1}x_1 + \ldots + a_{mn}x_n &= b_m. \end{aligned}$$

3. Define the rank of a matrix A and calculate the rank of

$$\begin{pmatrix} e^2 & \pi^2 & 9 \\ e & \pi & 3 \\ 2 \cdot e & 2 \cdot \pi & 6 \end{pmatrix}.$$

4. Prove that if $\{v_1, \ldots, v_n\}$ is a basis of a vector space V and $v = c_1v_1 + \ldots + c_nv_n$ is such that $c_1 \neq 0$ then $\{v, v_2, \ldots, v_n\}$ is a basis of V as well.

5. Prove or disprove that $\mathbb{R}^2$ with

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 + y_1 \\ x_2 + y_2 \end{pmatrix}, \quad c \cdot \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} c \cdot x_1 \\ x_2 \end{pmatrix},$$

where $x_1, x_2, y_1, y_2, c \in \mathbb{R}$, is a vector space.