

Sample Exam 2

Problem 1. Let $\mathbf{u}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\mathbf{u}_2 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$.

- Find the transition matrix S corresponding to the change of basis from $\{\mathbf{u}_1, \mathbf{u}_2\}$ to $\{\mathbf{v}_1, \mathbf{v}_2\}$.
- If $L : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear operator such that $L(\mathbf{v}_1) = \mathbf{v}_1 - 2\mathbf{v}_2$ and $L(\mathbf{v}_2) = -\mathbf{v}_1 + 2\mathbf{v}_2$, find the matrix representation of L with respect to the basis $\{\mathbf{v}_1, \mathbf{v}_2\}$.
- Find the kernel and image of L .
- Find the matrix representation of L with respect to the basis $\{\mathbf{u}_1, \mathbf{u}_2\}$.

Problem 2. Let $A = \begin{pmatrix} -2 & 0 & 0 \\ -5 & 3 & 0 \\ -5 & 2 & 1 \end{pmatrix}$.

- Find $\det A^5$.
- Find A^{-1} .
- If the map $L : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is given by $L(x) = Ax$ for any $x \in \mathbb{R}^3$, show that L is a linear transformation.
- Is L one-to-one? Is L onto? Is L an isomorphism?