

Solutions for Take-Home Quiz due 1/21

1. Find the solution set (in vector form, as in One.1.2) to the system of equations:

$$2x + 2y - 2z = 2$$

$$3x + 4y - 6z = 5$$

$$x + 3y - 7z = 5$$

Using matrix notation : $\left(\begin{array}{ccc|c} 2 & 2 & -1 & 2 \\ 3 & 4 & -6 & 5 \\ 1 & 3 & -7 & 5 \end{array} \right)$

Using Gauss-Jordan reduction takes several steps. I expect you to show your work along the way. To simplify the solution and keep track of all of these steps together leads to the following:

Replace ρ_1 with $-3\rho_1 + 3\rho_2 - 2\rho_3$,

replace ρ_2 with $\rho_1 - \rho_2 + \rho_3$,

and replace ρ_3 with $5\rho_1 - 4\rho_2 + 2\rho_3$.

(This demonstrates the lemma that linear combinations of linear combinations are linear combinations.)

This gives the reduced row-echelon form: $\left(\begin{array}{ccc|c} 1 & 0 & 2 & -1 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right)$

In equations, this matrix says:

$$x + 2z = -1$$

$$y - 3z = 2$$

$$0 = 0$$

The variables x and y are leading, and the variable z is free. The solution set consists of all vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ of the form } \begin{pmatrix} -1 - 2z \\ 2 + 3z \\ z \end{pmatrix} = \left\{ \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \cdot z \text{ such that } z \text{ is a real number} \right\}.$$

2. Write down the sentence from the Syllabus that starts with “Copying.”

There are two possible solutions to this. The most obvious one is to write “ Copying work to be submitted for a grade, or allowing your work to be submitted for a grade to be copied, is considered academic dishonesty.”

The second possibility is that you thought writing down this sentence was an act of copying. I disagree, but will still give full credit if you wrote something like: “This problem is a trick question. To write down the sentence here would be an act of copying work to be submitted for a grade—hence academic dishonesty. Thus the problem is a logical paradox and must be skipped.”