

## Solution for Quiz on 2/11

Find a basis and the dimension of the solution set of the following linear system of equations:

$$x + 2y + z + 3w + 4t = 0$$

$$2x + 4y + z + 6w + 6t = 0$$

$$3x + 6y + z + 9w + 8t = 0.$$

Reduce the coefficient matrix. 
$$\left( \begin{array}{ccccc|c} 1 & 2 & 1 & 3 & 4 & 0 \\ 2 & 4 & 1 & 6 & 6 & 0 \\ 3 & 6 & 1 & 9 & 8 & 0 \end{array} \right) \rightarrow \left( \begin{array}{ccccc|c} 1 & 2 & 0 & 3 & 2 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

The system has reduced to:

$$x + 2y + 3w + 2t = 0$$

$$z + 2t = 0.$$

Thus the solution set is: 
$$\left\{ \left( \begin{array}{c} -2y - 3w - 2t \\ y \\ -2t \\ w \\ t \end{array} \right) \mid y, w, t \in \mathbb{R} \right\}.$$

This set is three-dimensional and has basis 
$$\left\{ \left( \begin{array}{c} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{array} \right), \left( \begin{array}{c} -3 \\ 0 \\ 0 \\ 1 \\ 0 \end{array} \right), \left( \begin{array}{c} -2 \\ 0 \\ -2 \\ 0 \\ 1 \end{array} \right) \right\}.$$