Math 320

Quiz solutions: Monday, March 2nd

1. Is $f : M_{2\times 2} \to \mathbb{R}$, $f\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$ an isomorphism?

No, f is not an isomorphism because it is not one-to-one. Counterexample:
$f\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = f\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0.$

2. Is $f : \mathbb{R}^2 \to \mathbb{R}^3$, $f\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} a \\ 1 \\ b \end{pmatrix}$ a homomorphism?

No, f is not a homomorphism because it does not take the zero vector to the zero vector:
$f\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$