Math 294 Week 12

4/9/2019 or 4/11/2019

Given sets X and Y, we have the following relations:

Definition. We say that $X \subseteq Y$ if every element in X is also in Y.

Definition. We say that $X = Y$ if $X \subseteq Y$ and $Y \subseteq X$.

Definition. We also have the following set theoretic operations:

1. $X \cup Y = \{a \colon a \in X \text{ or } a \in Y\}$
2. $X \cap Y = \{a \colon a \in X \text{ and } a \in Y\}$
3. $X \times Y = \{(x, y) \colon x \in X \text{ and } y \in Y\}$
4. $\mathcal{P}(X) = \{A \colon A \subseteq X\}$

Definition. There's also a special set, called the empty set $\varnothing$. By definition, $\varnothing = \{\}$, the set with no elements. If we want to be really formal, we could say that $\varnothing = \{x \colon x \neq x\}$.

For submission, you must complete either 1 or 2, and also one of $3 - 6$.

Problem 1. True or false? Justify your answer.

1. $\varnothing = \{0\}$
2. $x \in \{x\}$
3. $\varnothing = \{\varnothing\}$
4. $\varnothing \in \varnothing$

Problem 2. List the elements of the following sets:

1. $\mathcal{P}(\varnothing)$
2. $\mathcal{P}(\{\varnothing\})$

3. $\mathcal{P}(\mathcal{P}(\varnothing))$
4. $\{\varnothing\} \times \mathcal{P}(\varnothing)$
5. $\varnothing \times \mathcal{P}(\varnothing)$
6. $\mathcal{P}(\varnothing) \times \mathcal{P}(\varnothing)$

Problem 3. Prove or disprove: If $A \cup B \subseteq A \cap B$, then $A = B$.

Problem 4. Let A and B be sets. Prove that $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.

Problem 5. Let A and B be sets. Prove or disprove that $A \times A = B \times B$ implies $A = B$.

Problem 6. Let A, B, and C be sets. Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. Draw a picture respresenting this.