Math 294 Week 14

4/23/2019 or 4/25/2019

Today we'll talk about limits!

Definition. A sequence $\{a_n\}$ in $\mathbb{R}$ converges to a if for every $\varepsilon > 0$ there's an $N \in \mathbb{N}$ such that, for each $n \geq N$, $|a_n - a| < \varepsilon$. This is written $a_n \to a$.

Remark. An important fact about absolute value signs is the triangle inequality, which says that $|a + b| \leq |a| + |b|$.

Definition. A sequence $\{a_n\}$ is bounded if there's an $M > 0$ such that $|a_n| \leq M$ for each $n \in \mathbb{N}$.

Problem 1. Prove the following:

- If $a_n \to a$, prove that $\{a_n\}$ is bounded.
- If $a_n \to 0$ and $\{b_n\}$ is a bounded sequence, prove that $a_n b_n \to 0$.
- Give a counterexample to the previous bullet when $\{b_n\}$ is unbounded.

Problem 2. If $a_n \to a$ and $b_n \to b$, prove the following:

- $\{a_n\}$ is bounded.
- $a_n + b_n \to a + b$
- $a_n b_n \to ab$. (Hint: $|a_n b_n - ab| = |a_n b_n - a_n b + a_n b - ab|$)

Problem 3. Let $\{a_n\} \to a$ and $\{b_n\} \to b$. Prove the following:

- If $a_n \geq 0$ for each n, then $a \geq 0$.
- $b_n - a_n \to b - a$
- Use the previous two bullets to prove that if $a_n \leq b_n$ for each $n \in \mathbb{N}$, then $a \leq b$.