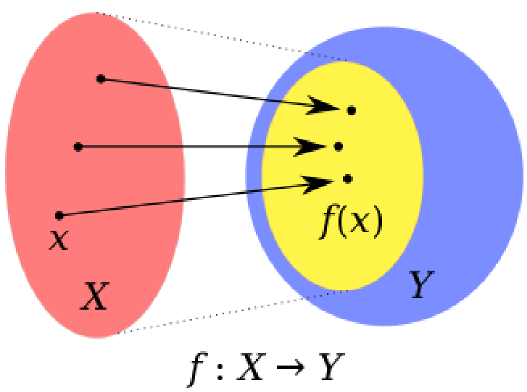
Math 294 Week 2

1/22/2019 or 1/24/2019

This worksheet will be emphasizing functions. Functions are arguably the most important concept in mathematics, and will appear in some form in every area of mathematics. Because functions appear so frequently, normally the most general definition will be expressed in terms of sets, which for us will just mean some collection of objects or numbers.

Definition. A function $f\colon X \to Y$ between sets X and Y is a rule or a mapping that assigns to each element $x \in X$ a unique element $y \in Y$. We write $y = f(x)$ if y was assigned to x.

A picture you might see fairly often is the following:



In the above definition, X is called the domain and Y is called the codomain. The range of a function is the set of all of the elements in Y assigned a value (i.e. the smaller yellow circle).

Definition. A function $f\colon X \to Y$ is:

1. injective (or 1-1) if for all $x, x' \in X$, $x \neq x'$ implies that $f(x) \neq f(x')$;
2. surjective (or onto) if for all $y \in Y$ there's an $x \in X$ such that $f(x) = y$;
3. bijective if it's both injective and surjective.

Problem 1. Explain why standard multiplication and addition of numbers is a function.

Problem 2. Let $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c\}$. Give an example of a function $f : A \to B$ that is neither injective nor surjective (one way to represent this is by drawing a picture like the one above).

Problem 3. A function $f : \mathbb{Z} \to \mathbb{Z}$ is defined as $f(n) = 2n+1$. Verify whether this function is injective and whether it is surjective and prove your answer.

Problem 4. Prove that if $f\colon A \to A$ is an injective function that's not surjective, then A must have infinitely many elements.

Problem 5. Prove that the function $f : \mathbb{R} \setminus \{2\} \to \mathbb{R} \setminus \{5\}$ defined by $f(x) = \frac{5x+1}{x-2}$ is bijective.

Problem 6. Prove the function $f : \mathbb{R}\setminus\{1\} \to \mathbb{R}\setminus\{1\}$ defined by $f(x) = (\frac{x+1}{x-1})^3$ is bijective.

Problem 7. A function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined as $f(m, n) = 3n - 4m$. Prove whether this function is injective and whether it is surjective.

Problem 8. Consider the function $h\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Q}$ defined as $h(m, n) = \frac{m}{|n|+1}$. Prove whether this function is injective and whether it is surjective.

Problem 9. Good luck! (Muhahahaha...)

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

AI14473 (c) Arto Inkala www.aisudoku.com

☆☆☆☆☆☆☆☆☆☆☆