Math 294 Week 5

2/12/2019 or 2/14/2019

This worksheet will emphasize truth tables and contrapostive proofs.

Problem 1. Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap. On trunk A is written: "At least one of these two trunks contains a treasure." On trunk B is written: "In A there's a fatal trap." Aladdin knows that either both the inscriptions are true, or they are both false. Can Aladdin choose a trunk being sure that he will find a treasure? If this is the case, which trunk should he open? Prove your answer either using truth tables or a proof by cases.

Problem 2. Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are:

- Box 1 "The gold is not here"
- Box 2 "The gold is not here"
- Box 3 "The gold is in Box 2"

Only one message is true; the other two are false. Which box has the gold? Formalize the puzzle in Propositional Logic and find the solution using a truth table.

Problem 3. You are walking in a labyrinth and all of a sudden you find yourself in front of three possible roads: the road on your left is paved with gold, the one in front of you is paved with marble, while the one on your right is made of small stones. Each street is protected by a guardian. You talk to the guardians and this is what they tell you:

- The guardian of the gold street: "This road will bring you straight to the center. Moreover, if the stones take you to the center, then also the marble takes you to the center."

- The guardian of the marble street: "Neither the gold nor the stones will take you to the center."
- The guardian of the stone street: "Follow the gold and you'll reach the center, follow the marble and you will be lost."

Given that you know that all the guardians are liars, can you choose a road being sure that it will lead you to the center of the labyrinth? If this is the case, which road you choose? Formalize the puzzle in Propositional Logic and find the solution using a truth table.

Prove the following statements with contrapositive proof. (In each case, think about how a direct proof would work. In most cases contrapositive is easier.)

Problem 4. Let $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

Problem 5. Let $k \in \mathbb{N}$. If $2^k - 1$ is prime, then k is prime. (Hint: use that $(a^n - 1) = (a-1)(a^{n-1} + \cdots + a^2 + a + 1)$ for any natural number n)